Banks, Liquidity Crises and Economic Growth*

Alejandro Gaytan
Banco de Mexico

Romain Ranciere
IMF Research Department

October 9, 2006

Abstract

How do the liquidity functions of banks affect investment and growth at different stages of economic development? How do financial fragility and the costs of banking crises evolve with the level of wealth of countries? We analyze these issues using an overlapping generations growth model where agents, who experience idiosyncratic liquidity shocks, can invest in a liquid storage technology or in a partially illiquid Cobb-Douglas technology. By pooling liquidity risk and reducing inefficient liquidation, a banking system delivers total factor productivity gains that allow a higher level of capital accumulation and growth. However, banks may face liquidity crises associated with severe output losses. We show that middle-income economies may find it optimal to be *exposed* to liquidity crises, while poor and rich economies have more incentives to develop a fully *covered* banking system. Therefore, middle-income economies could experience banking crises in the process of their development and, as they get richer, they eventually converge to a financially safe, long-run steady state.

JEL Classification No. G21, O16, O43. Key Words: Financial Intermediation, Banking Crises, Economic Growth, Economic Development, Volatility.

*We would like to thank Jess Benhabib, Fabio Braggion, Olivier Jeanne, Enisse Kharroubi, Boyan Jovanovic, Andrei Levchenko, Jaume Ventura, and especially Debraj Ray, as well as seminar participants at Bank of England, CREI, Pompeu Fabra, PSE, IMF, UC Irvine, NYU ,Lausanne and the participants to the CREI-Worldbank Conference on the Growth and Welfare Effects of Macroeconomic Volatility for useful comments. The views expressed in this paper are those of the authors and do not necessarily represent those of the Banco de México or the IMF.

1 Introduction

The development of a banking system to pool liquidity risk allows economies to achieve higher growth rates and higher long-run levels of wealth and consumption. However, a banking system may be vulnerable to liquidity crises with potentially large output and welfare losses in the short run. This paper investigates the relationship between the liquidity roles of banks, financial fragility and economic growth. It integrates the analysis of liquidity crises with the analysis of the long-run growth effects of financial intermediation. A novelty of this paper is that it develops a theory of the endogenous evolution of financial institutions along the path of economic development that can account for the observed differences in the financial vulnerability of banking systems and the associated costs of banking crises.

A large number of empirical studies supports the existence of a positive relationship between financial intermediation and growth. King and Levine [1995] and Levine et. al. [2000] find a positive effect of the relative size of the banking sector and several measures of financial development on per capita output growth.[1] There is also evidence that the contribution of financial development to growth is not uniform across countries. In particular, the growth-enhancing effect of financial intermediation has been shown to be larger for countries at some intermediate level of financial or economic development.[2]

In constrast to the growth literature, the banking crisis literature has pointed out the role of financial liberalization and the rapid increase in financial depth as good predictors of financial crises and their associated output losses.[3] Reconciling these two views, Loayza and Ranciere [2005] show that for some countries a positive long-run relationship between financial intermediation and output growth can coexist with a negative short-run relationship, especially for countries that have suffered episodes of financial crises.

Table 1 presents information on the level of income per capita and the number of banking crises.[4] Countries are divided into quartiles according to their level of GDP per capita. The table shows that banking crises are more frequent in middle-income countries than in low-income and high-income countries. Moreover, emerging economies have not only experienced a higher recurrence of banking crises but have also incurred more severe costs. This is shown in Figure 1, which plots the

[1] Benhabib and Spiegel [2000] show that both total factor productivity growth and factor accumulation are stimulated by financial development.

[2] See, for instance, Aghion, Howitt and Mayer [2003], Gaytan and Ranciere [2005] and Rioja and Valed [2003]

[3] See, for example, Demirguc-Kunt and Detragiache [1998 and 2000]; Gourinchas, Landerretche and Valdes [1999]; Kaminsky and Reinhart, [1999].

[4] Caprio and Klingebiel [2003] define a systemic banking crisis as a situation where aggregate capital of the banking sector has been exhausted.

cumulative fiscal cost of banking crises (as a percentage of GDP) for countries ranked according to their average income per capita. Banking crises have been much more severe for middle-income economies than for poor or rich economies.

This paper focuses on the allocative and liquidity functions of banks. In particular, financial intermediaries *(i)* provide an efficient mechanism that channels savings into those investments with the highest returns, *(ii)* are efficient suppliers of liquidity (i.e. they transform illiquid assets into liquid liabilities), and *(iii)* provide liquidity insurance that eliminates idiosyncratic liquidity risk.[5] We study the costs and benefits of these three liquidity functions for welfare and growth, and how they change during the process of economic development. In particular, we focus on banking crises caused by liquidity mismatches, leaving aside the role of asymmetric information associated with bank lending.

We use an intertemporal model of financial intermediaries to analyze the optimal deposit contract and investment portfolio and their implications for the dynamics of wealth, capital and consumption. The model embeds a modified version of the Diamond and Dybvig [1983] model of liquidity provision (henceforth DD) into an overlapping generations model (Diamond 1965). Two technologies are available: a short-term storage technology and a long-term partially illiquid technology. In this paper, the long-term technology uses a standard Cobb-Douglas production function with labor and capital as inputs. This technology constitutes the channel for growth over time and across generations.The level of total factor productivity is endogenous and depends on the fraction of long-term projects that are prematurely liquidated.

We characterize the optimal deposit contract offered by a competitive bank when panic runs can occur with positive probability, and we show how this contract changes with the level of wealth of the economy.[6] Whenever bank runs are possible, the design of the contract offered by the bank involves a decision between being covered (i.e. protected against runs) and taking the risk of being exposed to liquidity crises. Under *covered banking*, the bank suppresses any incentive for agents to run by designing a demand deposit contract and choosing an investment portfolio that ensures that depositors' claims will always be satisfied. Under *exposed banking*, the bank will face a liquidity crisis if all agents attempt to prematurely withdraw their funds. Covered banking is possible at the cost of lower liquidity insurance, while exposed banking has the cost of possible crisis episodes. For any given level of wealth and unconditional probability of crisis, an *optimal banking system*

[5]Other functions of banks studied in the literature on financial intermediation and growth include the pooling of risk among different investment projects and the banks' ability to mitigate information frictions (see, for example, Saint-Paul 1992, Greenwood and Jovanovic 1990).

[6]Cooper and Ross [1998] have shown that the original Diamond-Dybvig solution does not consider the impact of the possibility of runs on the design of the optimal deposit contract or the bank's investment portfolio.

will choose the banking arrangement (covered or exposed) that maximizes the welfare of current depositors.

The characterization of the optimal banking system constitutes the key result of this paper. For a sufficiently high unconditional probability of crisis, a covered banking system would be optimal for any level of wealth; for lower probabilities, poor and rich economies would opt for a covered banking system, while middle-income economies would choose an exposed banking system. Finally, when the risk of experiencing a run is small enough, poor and middle-income economies will choose to be exposed to liquidity crises. Nevertheless, as long as the probability of runs is positive, there will be a level of wealth above which a covered banking system would be optimal. In other words, economies that choose an exposed banking system take on the risk of banking crises with short-run output losses in exchange for higher liquidity insurance and possibly higher returns and growth. Nevertheless, as they get richer, they can eventually "escape" financial vulnerability and converge to a financially safe long-run steady state.

Comparing the nature of the growth process under financial autarky and under financial intermediation reveals an important implication of an optimal banking system. Under financial autarky, the growth process is entirely neo-classical and driven by decreasing marginal returns to capital. Because of a constant and inefficient level of liquidation of long-term projects, the economy is trapped at a low level of total factor productivity. By contrast, under financial intermediation, the economy evolves through two distinct growth regimes: an *endogenous growth regime* and a *neo-classical regime*. Under the endogenous growth regime, the reduction of inefficient liquidation achieved by the banking system is an endogenous source of total factor productivity growth. The increase in total factor productivity allows for higher investment and growth in the neo-classical regime that follows. This finding is consistent with the empirical evidence of an important increase in the contribution of financial development to economic growth at some intermediate stage of economic development.[7] The endogenous regime can be interpreted as a *take-off* phase during which the evolution of the financial system generates a permanent increase in the productivity of investment.

Another consequence of financial intermediation is that it generates growth volatility whenever an exposed banking system is optimal. In this case, the impact of a banking crisis on growth depends on the severity of the output losses. We show that output losses in the case of a crisis are more severe for middle income economies than for rich economies.

Previous literature has studied liquidity provision by financial intermediaries in an intertemporal framework. In particular, Bencivenga and Smith [1991] embed the DD model into an overlapping

[7] See Rioja and Valed [2003] and Gaytan and Ranciere [2005]

generations framework to analyze the link between financial intermediation and growth .[8] Ennis and Keister [2003] extend Bencivenga and Smith [1991] in order to analyze the growth effects of banking crises. Their paper is the closest to ours. However, a key difference to our paper is that Ennis and Keister [2003] and Bencivenga and Smith [1991] consider an endogenous growth model with constant returns to capital. Under this assumption, and in contrast with the empirical evidence, the growth effects of financial intermediation as well as financial fragility are independent of the level of economic development. By contrast, in our model, decreasing returns to long term investment make the optimal banking solution dependent on the economy's level of wealth.

The remainder of the paper is organized as follows. Section 2 describes the general set-up of the model. Section 3 characterizes the optimal banking system. Section 4 analyzes the growth consequences of an optimal banking system and discusses the output losses of banking crises. Section 5 concludes.

2 The Model

2.1 The basic set-up

The economy consists of an infinite sequence of overlapping generations. In each period, a generation, composed of a continuum of ex-ante identical agents with unit mass, is born. There is no population growth. There is a single good, used for consumption and investment.

Agents. Agents live for two periods. They have an endowment of one unit of labor during the first period of their lives, which they supply inelastically. Agents do not value consumption when they are young. During the second period of their life they are subject to a time preference shock. With probability π an agent only values consumption when middle-aged (the middle of her second period), and becomes an *early consumer.* With probability $(1-\pi)$ she only values consumption when old (the end of her second period) and becomes a *late consumer.* This liquidity shock is stochastically independent across time and agents, and is private information to the agent.Therefore, preferences of an agent that belongs to generation t are*:*

$$U\left(c_E^t, c_L^t\right) = \Gamma u\left(c_E^t\right) + (1-\Gamma)u(c_L^t) \tag{1}$$

$$with\ \Gamma = \begin{cases} 1 & \text{with probability} \quad \pi \\ 0 & \text{with probability} \quad (1-\pi) \end{cases}$$

[8] Qi [1994] and Fulghieri and Rovelli [1998] also study the DD model in an overlapping generations framework. However, their focus is on intergenerational transfers in an endowment economy and not on economic growth.

where $c_E^t, c_L^t \geq 0$ are the levels of early and late consumption, respectively, of an agent born at t, and $u(\cdot)$ belongs to the constant relative risk aversion class of utility functions: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$.

Risk averse agents would like to reduce the *ex-ante* gap between early and late consumption. The level of *liquidity insurance* is defined by the marginal rate of subsitution between late and early consumption and is equal to $\frac{u'(c_L)}{u'(c_E)} = \left(\frac{c_E}{c_L}\right)^{\sigma}$.[9]

Technologies. Two technologies are available. The first technology is a storage technology that uses the good as the only input. Each unit invested at t yields a constant return, normalized to one unit, in any sub-period of $t+1$. There is also a long-term Cobb-Douglas technology which uses labor l and capital k as inputs.[10] This technology can be left until full maturity or can be prematurely liquidated in the middle of the period. In the latter case it yields only a fraction γ of the return at maturity:[11]

$$G(k,l) = \begin{cases} F(k,l) = Ak^{\beta}l^{1-\beta} & \text{if liquidated at } t+1 \\ \gamma F(k,l) = \gamma Ak^{\beta}l^{1-\beta} & \text{if liquidated at } t+\frac{1}{2} \end{cases}$$

with $0 < \gamma < 1$

Since the unit of labor is supplied inelastically, we can write the capital intensive production function as:

$$f(k) \equiv F(k,1) = Ak^{\beta}$$

Throughout the paper, it is convenient to express the return to investment in the long-term technology as:

$$h(k) \equiv f'(k)k = \beta f(k)$$

Factor Markets. Factor markets are competitive, so each factor is paid its marginal product. The realization of the marginal product depends on the proportion of long term projects liquidated, which in turn depends on the financial arrangement in place.

[9] Given the CRRA preferences, the level of liquidity insurance $\left\{\frac{u'(c_L)}{u'(c_E)} = \left(\frac{c_E}{c_L}\right)^{\sigma}\right\}$ is a simple power transformation of the ratio of early to late consumption $\left(\frac{c_E}{c_L}\right)$.

[10] For simplicity it is assumed that capital fully depreciates after being used in production.

[11] An example illustrates this long-term technology. We can think about this technology as a crop. It is irreversible in terms of the original capital invested (seeds). If it is left until full maturity, it yields the maximum size of the crop; however, premature liquidation would yield a crop that is not fully grown. Finally, the amount of labor required both at the planting and at the harvest is the same, and it is independent of the timing of the harvest.

Wages are paid at the end of each period and are the unique income source of young agents. After receiving wages, agents make investment decisions before observing the realization of their liquidity shock. Since agents do not value consumption when young, their consumption-saving decision is trivial, and they invest their wealth either directly in the two technologies (under financial autarky) or as bank deposits (under financial intermediation).[12] Figure 2 summarizes the timing of the economy.

The Financial Arrangement. The basic set-up of the economy is completed by the financial arrangement in place. In this paper we consider two financial arrangements: a benchmark case of financial autarky and the case of financial intermediation.

Financial autarky provides a useful benchmark to assess the welfare and growth costs and benefits of financial intermediation. In this environment, agents invest directly in the two technologies available and, in the absence of a mechanism to share liquidity risk, the investment decision reflects their need to self-insure against future liquidity shocks. The derivation of the optimal investment policy under autarky and the analysis of its growth implications are summarized in Appendix A.

For the *financial intermediation* arrangement, we consider a generational bank that pools all deposits and maximizes expected utility of current depositors, which is equivalent to a profit maximizing competitive banking system. The bank chooses the investment portfolio and provides liquidity and liquidity insurance to depositors through a demand deposit contract. Under this arrangement, the idiosyncratic liquidity shock is private information to the agent, and the bank has to offer incentive-compatible allocations. However, even when a truth revelation mechanism is in place, panic bank runs are still possible, and the optimal demand deposit contract must consider the bank's expectations about the probability of a panic.

Discussion of the set-up

The assumptions about the technologies imply a trade-off between liquidity and return that evolves with the level of wealth of the economy. Let us define the two following threshold levels of capital:

$$\underline{k} \text{ such that } \gamma h'(\underline{k}) = 1$$
$$\overline{k} \text{ such that } h'(\overline{k}) = 1$$

Since labor is supplied inelastically, the long-term technology presents diminishing marginal returns to capital. Figure 3 describes the marginal returns of the technologies as functions of the

[12] We abstract from the consumption-saving decision to stress the choice among assets with different liquidity.

level of investment. For any level of investment in the range $(\underline{k}, \overline{k})$, there is a trade-off between liquidity and return as $\gamma h'(k) < 1 < h'(k)$.[13] Furthermore, as investment in the long-term technology increases, its marginal return decreases and the trade-off between liquidity and returns becomes gradually more favorable to the liquid technology.[14]

The timing of the economy implies that at any point in time, only one generation of investors is alive. Hence, the banking system is formed by successive generational banks, making the problem of the design of the deposit contract static. This simplification is standard in growth models embedding two-period financial contracts.[15] It has the advantage of keeping the model tractable enough to allow for a complete characterization of the transitional dynamics of the economy.[16]

3 The Optimal Banking System

All liquidity uncertainty in this economy pertains to the liquidity needs of individuals, and it is idiosyncratic. Under financial autaky (see Appendix A), agents have no access to a financial system to pool or trade their risk, and the result is a mismatch between ex-post liquidity needs of the agents and the timing of highest returns of the assets, generating an inefficient aggregate liquidation of the long-term projects. Financial intermediation can improve welfare by providing risk sharing and by delivering an efficient balance between the agents' preference for liquidity insurance and the timing of the highest returns on the assets.

However, since liquidation is costly, if the value of the bank's assets at the early sub-period cannot cover a total withdrawal of deposits, the bank is vulnerable to a panic run. A financial crisis driven by a panic appears as a coordination problem in which late consumers believe that the bank won't be able to service all deposits in the late sub-period, driving a total run on the bank at the beginning of $t+1$. The optimal deposit contract is influenced by the possibility of a financial panic. The bank faces a tension between improving the welfare of depositors by offering higher

[13] For low levels of capital investment ($k < \underline{k}$), the marginal return of the long-term technology, even when prematurely liquidated, exceeds the marginal return of the storage technology ($\gamma h'(k) \geq 1$). Beyond some level of investment in the long asset ($k > \overline{k}$), its marginal return at full maturity is smaller than one ($h'(k) \leq 1$). Hence, we can anticipate that irrespective of the financial environment, optimal investment in the long-term technology (i) will not exceed $\overline{k}$ for any level of wealth, and (ii) will be equal or larger than $\underline{k}$, as long as wealth is greater than $\underline{k}$.

[14] The assumption that the liquid technology has constant marginal returns and that the long-term technology is concave is analytically convenient. The results of the paper are robust to a broader range of specifications regarding the concavity of the two technologies, as long as they imply a trade-off between liquidity and return.

[15] For example, Aghion et. al. [2005] use a similar assumption to include, in a tractable way, a version of the Holmstrom-Tirole liquidity model [1998] in an OLG growth model.

[16] When intergenerational banks are considered, the analytical solution is generally restricted to the analysis of the steady state (See Qui 1994 and Fulgheri and Rovelli 1998)

returns and liquidity insurance and having a more vulnerable system. If the bank could assign a probability to the event of a financial panic, it could find the most efficient balance between these two objectives.

3.1 Generation t Optimal Risk-Sharing

The generational bank pools labor income of agents (w). By the law of large numbers, all liquidity uncertainty disappears since the bank knows that a proportion π of agents will need their deposits in the early sub-period, and a proportion $(1-\pi)$ in the late sub-period. Therefore, the bank can offer a deposit contract that promises a fixed payment c_E at the beginning of period $t+1$, and c_L at the late sub-period of $t+1$. To provide the optimal risk sharing contract the financial intermediary chooses the investment portfolio k and the optimal liquidation policy. The liquidation policy is used to transfer resources between sub-periods, and it is composed of two parts: the bank can liquidate a proportion λ of the long-term technology to serve early consumers; in addition, it might be optimal to keep in storage an amount i of the short asset, or "excess liquidity", for late consumption. The deposit contract, investment portfolio and liquidation policy are aimed to form the most efficient match between the liquidity needs of agents and the highest returns of the assets. Since the type of agent remains private information, the optimal solution of the bank is constrained by a self revelation mechanism: an incentive compatible contract must offer higher consumption in the late sub-period $(c_E \leq c_L)$, so that patient agents have an incentive to wait until the full realization of the assets' returns.

Existence of a Bank Run Equilibrium

At the beginning of $t+1$ those agents that claim to be early consumers withdraw their deposits, and the bank is forced to liquidate any amount of assets required to satisfy their demand. In the second sub-period, those agents that waited receive an equal share of the remaining assets of the bank. When all agents withdraw their deposits according with their type, the aggregate demand for early withdrawals is πc_E, while if all late agents misrepresnt their type, this demand is c_E. Late agents face the decision of waiting and receiving a share of the remaining assets, if there are any left, or withdraw early. Costly liquidation of the long technology implies that the value of the bank's total portfolio at the early sub-period $(w-k+\gamma h(k))$ is lower than its value when the technologies are left to mature as planned $w-k+(\lambda\gamma+1-\lambda)h(k)$. Therefore, a run strategy may only be optimal if the value of all liabilities in the early sub-period exceed the liquidation value of the banks portfolio, that is, if:

$$c_E > c_R \equiv w - k + \gamma h(k) \tag{2}$$

When (2) holds, there are two possible equilibria: an *honest equilibrium* where agents withdraw from the bank according with their true type, and a *run equilibrium* where all agents withdraw their deposits, pretending to be early consumers. In the run equilibrium the bank declares bankruptcy and distributes the value of its assets among claimants following a bankruptcy rule. We assume that the bank has to give the same amount to consumers reporting to the bank at the same time. In case of a run, there is a pro-rata distribution of assets, and the bank provides all consumers an equal share c_R[17].

Equilibrium Selection Mechanism.

A maximizing bank must necessarily realize that a contract for which (2) holds makes it vulnerable to bank runs, and this fact will affect the design of the contract. The question of how the equilibrium is selected when both equilibria are possible is crucial to determine how it affects the choice of the optimal contract. In the absence of additional uncertainty, it is not clear what drives expectations about the future solvency of the bank. In this paper we assume the most basic equilibrium selection mechanism: a sunspot.[18] We assume that there is a publicly observable variable that influences the agents' level of "optimism" about the solvency of the bank. With probability q this variable takes values that lead to a pessimistic assessment about future solvency. Nevertheless, pessimistic expectations can lead to a financial crisis only when the bank is vulnerable.

3.1.1 The Bank's Problem

Let $\theta \in \{0, 1\}$ be the state variable of a bank run. If $\theta = 1$, late agents withdraw the deposits in the early sub-period, and if $\theta = 0$, all agents make their withdrawals according to their type. Let η be the probability of a bank run conditional on the optimal contract and investment portfolio. If the contract makes the bank solvent under any circumstance ($c_E \leq c_R$), a bank run is never optimal and the conditional probability is zero ($\eta = 0$). In contrast, if (2) holds, the probability of a bank run is the probability of pessimistic expectations ($\eta = q$).

At any period t, and for any given level of deposits (wealth $w > 0$), a representative bank of gen-

[17] In the honest equilibrium, agents don't care about their position in the bank line, as there are enough assets to serve them all the promised amount c_E. By contrast, in case of a run, all agents want to be "first in line" and thus will show up at the bank at the same time. The example of the recent run on Argentinian banks in 2002 is illustrative: all agents who were waiting in front of the bank before the opening were allowed to withdraw an equal fraction of their deposits.

[18] Several authors have studied bank runs as an equilibrium phenomenon (Postlwaite and Vives [1987], Jacklin and Bhattacharya [1988], Cooper and Ross [1998], Allen and Gale [1998]). These papers either assume an exogenous probability of crises, or neglect the possibility of panic-based runs. Goldstein and Pauzner [2005] tackle the problem of equilibrium selection and endogenize the probability of bank runs. Based on the ideas of global games, developed by Carlsson and van Damme [1993] and Morris and Shin [1998], the authors show that the existence of aggregate uncertainty and imperfect and asymmetric private information can select a unique equilibrium in the static DD model.

eration t-depositors chooses k, λ, i, c_E, and c_L to maximize the expected utility of a representative depositor:[19]

$$V(\eta, w) = \max_{k,\lambda,i,c_E,c_L} (1-\eta)\left[\pi u(c_E) + (1-\pi)u(c_L)\right] + \eta u(c_R) \quad \text{subject to:} \tag{P0}$$

$$\pi c_E \leq w - k - i + \lambda\gamma h(k) \tag{3}$$

$$(1-\pi)c_L + \pi c_E \leq w - k + \lambda\gamma h(k) + (1-\lambda)h(k) \tag{4}$$

$$c_E \leq c_L \tag{5}$$

$$0 \leq \lambda \leq 1 \tag{6}$$

$$0 \leq k \leq w \tag{7}$$

$$0 \leq i \leq w - k \tag{8}$$

$$c_R = w - k + \gamma h(k) \tag{9}$$

$$\eta = \begin{cases} \Pr(\theta = 1 | k^*, \lambda^*, i^*, c_E^*, c_L^*) = 0 & \Leftrightarrow c_E \leq c_R \\ \Pr(\theta = 1 | k^*, \lambda^*, i^*, c_E^*, c_L^*) = q & \Leftrightarrow c_E > c_R \end{cases} \tag{10}$$

Equation (3) is the resource constraint at the early sub-period of $t+1$; for serving agents with early liquidity needs, the bank can use part of the short asset $(w-k-i)$ and a proportion λ of the long-term technology. Equation (4) is the resource constraint at the late sub-period of $t+1$; the bank uses all its remaining assets to serve late consumers. Equation (5) is the incentive compatibility constraint. Finally, the probability of a bank run (10) given the optimal contract is equal to the sunspot probability if the bank is vulnerable to a crisis and zero otherwise.

It is useful to decompose the bank's problem into two decision problems taken at different stages. The bank can offer two alternative types of contracts. Under the first type of contract, termed *"covered banking,"* the financial intermediary chooses a contract that makes it invulnerable to crisis ($c_E \leq c_R \Rightarrow \eta = 0$). In this case, the returns on deposits are independent of the realization of the sunspot. Under the second type of contract, termed *"exposed banking,"* the bank takes on the risk of having a run on its deposits ($c_E > c_R \Rightarrow \eta = q$). Thus, in the first stage, the bank determines the optimal exposed and the optimal covered contracts. In the second stage, the bank selects between these two contracts the one that maximizes expected utility.

[19] The bank centralizes production and pays a wage to the following generation (w') equal to the realized marginal product of labor $w' = (1-\beta)(\lambda\gamma + 1 - \lambda)f(k)$.

The optimal covered banking contract $O^c = \{k^c, \lambda^c, i^c, c_E^c, c_L^c\}$ solves the problem:

$$V^c(w) = \max_{k,\lambda,i,c_E,c_L} \pi u(c_E) + (1-\pi)u(c_L) \text{ subject to:} \tag{P^s}$$

$$(3), (4), (5), (6), (7), (8), \text{ and}$$

$$c_E \leq w - k + \gamma h(k) \tag{11}$$

where (11) is the run-preventive constraint.

The optimal exposed banking contract $O^e = \{k^e, \lambda^e, i^e, c_E^e, c_L^e\}$ solves the problem:

$$V^e(q, w) = \max_{k,\lambda,i,c_E,c_L} (1-q)\left[\pi u(c_E) + (1-\pi)u(c_L)\right] + qu(c_R) \text{ subject to:} \tag{P^c}$$

$$(3), (4), (5), (6), (7), (8), \text{ and } (9)$$

In the second stage of the problem, the bank chooses the contract that gives the larger expected utility, which is equivalent to choose $\eta^* = \arg\max \{V(\eta, w)\}$, where $V(\eta, w) = Max\{V^e(q, w), V^c(w)\}$. The optimal banking contract $O^* = \{k^*, \lambda^*, i^*, c_E^*, c_L^*\}$ and the associated level of expected utility $V^*(\eta^*, w)$ define the optimal banking solution also referred to as the *optimal banking system.*

The analysis of tensions and distortions in the optimal contract generated by the possibility of crises requires the definition of an efficient benchmark. We consider the intra-generational first-best solution, in which a planner can observe the realization of the idiosyncratic liquidity shocks. We will refer to this benchmark as the *first best* or *unconstrained optimal risk sharing solution.*

The optimal unconstrained risk-sharing contract $O^u = \{k^u, \lambda^u, i^u, c_E^u, c_L^u\}$ solves the problem:

$$V^u(w) = \max_{k,\lambda,i,c_E,c_L} \pi u(c_E) + (1-\pi)u(c_L) \text{ subject to:} \tag{P^u}$$

$$(3), (4), (6), (7), (8)$$

Two preliminary remarks are in order. First, the first best solution weakly dominates the optimal banking solution.[20] Second, the first best contract corresponds to the optimal exposed contract in the limiting case where the probability of run tends to zero.[21]

[20] The optimal banking solution is a feasible solution in the unconstrained risk-sharing program P^u.

[21] This is the case because the first best contract always satisfies $c_E \leq c_L$. The first best solution also corresponds to the original risk-sharing solution in Diamond and Dybvig [1983], since, as first noted by Cooper and Ross [1998], the design of the DD contract does not internalize the possibility of bank runs.

The General Shape of the Solutions.

General expressions for the optimal early and late payoffs for all contracts and regions are given by:

$$c_E^j = \frac{w - k^j - i^j + \lambda^j \gamma h\left(k^j\right)}{\pi} \qquad and \qquad c_L^j = \frac{i^j + \left(1 - \lambda^j\right) h\left(k^j\right)}{1 - \pi}, \tag{12}$$

where $j = \{u, c, e\}$ indexes the unconstrained or first-best solution, the covered banking solution, and the exposed banking solution, respectively.

Before presenting the first-best, covered and exposed contracts, it is possible to characterize the general shape of the solutions. In general, the optimal solution defines four regions (**A** to **D**) dependent on the level of wealth. The wealth thresholds that define these regions are $\underline{k},\ \widetilde{w}^j,\ and\ \hat{w}^j$ for $j \in \{u, c, e\}$.

Region A: No investment in short-term technology, no liquidity provision.

For low levels of wealth ($w \leq \underline{k}$), the marginal return of the long-term technology, even if it is liquidated, is higher than the return of the short-term technology. Therefore, all wealth is invested in capital ($k = w$), and early consumption is served by liquidating a constant proportion of its asset (constant λ^j).

Region B: Constant level of investment in capital, reduction of early liquidation, increasing liquidity provision.

For $\underline{k} \ \leq w \leq \widetilde{w}^j$, the financial intermediary invests in both assets and provides extra liquidity. The main characteristic of this region is that investment in capital is kept fixed at a level $\underline{k}$ that equates the marginal return of the long-term technology, when liquidated prematurely, to the return of the short-term technology. In this region, the bank starts using the liquid technology as a source of liquidity to pay out early consumers, reducing premature liquidation of the long-term technology (decreasing λ^j).

Region C: No liquidation of long term investment, increasing investment in both assets.

When wealth has crossed a certain threshold ($w \geq \widetilde{w}^j$), the financial intermediary stops using the long-term technology to serve early consumers ($\lambda^j = 0$). Early consumers are served only using the short-term technology ($c_E^j = \frac{w - k^j}{\pi}$), and late consumers using the long term technology ($c_L^j = \frac{h\left(k^j\right)}{1-\pi}$). As wealth increases over this region, investment in both technologies increase.

Region D: No liquidation of long term investment, excess liquidity.

For high levels of wealth ($w > \hat{w}^j$), high investment in the long term technology has exhausted the marginal return of capital, and it becomes optimal to use the proceeds of the short-term technology to serve late consumers ($i^j \geq 0$).

3.1.2 The Efficient Benchmark: Unconstrained Optimal Risk Sharing

In the following table, we characterize the investment portfolio, the liquidation policy and the liquidity insurance implied by the unconstrained optimal risk-sharing solution.[22]

	Wealth: w	Capital: k^u	Liquidation: λ^u	Excess Liquidity: i^u	Liquidity Insurance: $\left(\frac{c_E}{c_L}\right)^{\sigma}$
A	$0 < w \leq \underline{k}$	w	$\lambda^* \equiv \frac{\pi\gamma^{\frac{1}{\sigma}}}{\pi\gamma^{\frac{1}{\sigma}}+(1-\pi)\gamma}$	0	γ
B	$\underline{k} \leq w \leq \widetilde{w}^u$	$\underline{k}$	$\lambda^* - (1-\lambda^*)\,\beta\frac{w-\underline{k}}{\underline{k}}$	0	γ
C	$\widetilde{w}^u \leq w \leq \hat{w}^u$	$k^u(w)$	0	0	$h'(k^u)^{-1}$
D	$w \geq \hat{w}^u$	$\overline{k}$	0	$(1-\pi)(w-\overline{k}) - h(\overline{k})$	1

where $k^u(w)$ in region C is strictly increasing in the level of wealth.[23]

The unconstrained solution provides a benchmark to evaluate the efficiency of the covered and exposed banking solutions in three dimensions: technological efficiency, efficiency in the provision of liquidity, and efficiency in the provision of insurance against idiosyncratic shocks.

Technological efficiency requires that: (i) there is full investment in capital if and only if the marginal return of the liquidated long-term technology exceeds the return of the short-run technology ($k = w \Leftrightarrow \gamma h'(k) \geq 1$); (ii) as long as the bank liquidates part of long-term technology to serve early consumers, investment in capital never exceeds $\underline{k}$ ($\lambda > 0 \Rightarrow k < \underline{k}$); (iii) for large enough levels of wealth $\left(w \geq \hat{w}^j\right)$, the bank fully exploits the return of the long-term technology by investing $\overline{k}$.[24]

Efficiency in the provision of liquidity entails that: (i) long-term projects are liquidated only when their liquidation yields a marginal return higher than the return to the short technology ($\gamma h'(k) \leq 1 \Rightarrow \lambda = 0$); and (ii) there is no excess liquidity ($i = 0$) as long as the marginal return of the long-term technology at maturity exceeds the return of the short technology ($h'(k) > 1 \Rightarrow i = 0$).

Efficiency in the provision of insurance is achieved when the marginal rate of substitution $\left(\frac{u'(c_L)}{u'(c_E)}\right)$ is equal to the ratio of the marginal returns of the technologies used to serve early and late consumers. This property implies that: (i) when both technologies are used, an increase in

[22] The derivation of the optimal risk-sharing solution is included in the supplemental appendix.

[23] $k^u(w)$ is uniquely defined by the f.o.c.: $\frac{u'(c_E)}{u'(c_L)} = \left(\frac{\pi h(k^u(w))}{(1-\pi)(w-k^u(w))}\right)^{\sigma} = h'(k^u(w))$. The wealth thresholds are given by: $\widetilde{w}^u = \underline{k}\left(1 + \frac{\pi\gamma^{1/\sigma}}{(1-\pi)\gamma\beta}\right)$, $\hat{w}^u = \overline{k}\left(1 + \frac{\pi}{\beta(1-\pi)}\right)$

[24] By the definition of $\underline{k}$ and $\overline{k}$: $k \leq \underline{k} \Leftrightarrow \gamma h'(k) \geq 1$, $k = \underline{k} \Leftrightarrow \gamma h'(\underline{k}) = 1$ and $h'\left(\overline{k}\right) = 1$

invesment in the long techology must be associated with an increase in the provision of liquidity insurance ($k < w$ and k increasing$\Rightarrow \frac{c_E}{c_L}$ increasing); and (ii) excess liquidity is held ($i > 0$) only when perfect insurance is provided.

An important question is whether a bank that implements the unconstrained risk-sharing contract presented above is exposed to bank runs. If it is not the case, the optimal banking system and the first best solution must coincide.[25] The following proposition shows under which conditions the optimal banking system achieves the first best.

Proposition 3.1 (Optimal Banking System and Unconstrained Optimal Risk-Sharing) *The optimal banking system attains the unconstrained risk-sharing solution (first best) if and only if risk aversion is no greater than unity ($\sigma \leq 1$) and wealth is low enough ($w < w_{rp}$),*

where $w_{rp} = k_{rp}(1 + \frac{\pi\gamma}{(1-\pi)\beta\gamma^{\sigma}})$ and $h'\left(k_{rp}\right) = \frac{1}{\gamma^{\sigma}}$.

Proof. See Appendix B. ■

Impatient agents ($\sigma > 1$) have a strong preference for liquidity insurance and demand higher early payoffs, making the first-best contract vulnerable to runs. In this case, the possibility of crises prevents that the optimal banking system attains the first best solution. Patient agents ($\sigma \leq 1$), on the other hand, prefer to enjoy higher payoffs on late withdrawals while the marginal returns on long-term projects are still high and the optimal banking system can achieve the unconstrained solution. Nevertheless, since liquid insurance increases with the level of wealth, the first best contract eventually becomes vulnerable to runs.[26]

For $0 < w \leq w_{rp}$ and $\sigma \leq 1$, the optimal banking solution is covered banking since it delivers the first best solution. For higher levels of income, the optimal contracts are subject to the optimality conditions that prevail for $\sigma > 1$. Therefore, in what follows, we concentrate our attention on the results for high risk aversion ($\sigma > 1$).[27]

[25] The argument goes as follows: (i) if the unconstrained risk-sharing contract respects the run-preventive constraint, it must be the optimal covered contract; (ii) if the run-preventing constraint is not binding in the optimal covered contract, insurance against crisis does not entail any welfare cost and thus covered banking is the optimal banking system.

[26] Improving insurance and the existence of a wealth level above which the economy is vulnerable to a run represent a difference with respect to the original DD model. In their original framework of fixed returns to assets, low risk aversion ($\sigma \leq 1$) implied that the optimal risk sharing contract was necessarily run proof.

[27] The case $\sigma > 1$ is also the most relevant case since the plausible values for σ considered in business cycle and growth literature typically range between 1 and 10.

3.1.3 Covered Banking ($\eta = 0$).

Before presenting the optimal covered contract, it is useful to notice that the autarkic solution is run proof ($c_E^a = w - k + \pi\gamma h(k) < w - k + \gamma h(k)$). Since a covered bank could always replicate the autarkic solution, optimal covered banking weakly dominates the autarkic solution for any level of wealth, and strongly dominates autarky for $w > \underline{k}$.[28]

Proposition 3.2 *The optimal covered banking contract for high risk aversion ($\sigma > 1$) is characterized by the following conditions:*

	Wealth: w	Capital: k^c	Liquidation: λ^c	Excess Liquidity: i^c	Liquidity Insurance: $\left(\frac{c_E}{c_L}\right)^\sigma$
A	$0 < w \leq \underline{k}$	w	π	0	γ^σ
B	$\underline{k} \leq w \leq \widetilde{w}^c$	$\underline{k}$	$\pi - (1-\pi)\beta\frac{w-\underline{k}}{\underline{k}},$	0	γ^σ
C	$\widetilde{w}^c \leq w \leq \hat{w}^c$	$k_C^c(w)$	0	0	γ^σ
D	$w \geq \hat{w}^c$	$k_D^c(w)$	0	$(1-\pi)(w-k^c) - \pi\gamma h(k^c)$	$\frac{\pi}{1-\pi}\frac{(1-\gamma h'(k))}{\left[\left(\frac{1-\pi\gamma}{1-\pi}\right)h'(k)-1\right]}$

where $k_C^c(w)$ and $k_D^c(w)$ are strictly increasing in the level of wealth.[29] Figure 4a illustrates the optimal choice of capital and liquidity insurance for a simulation of the economy.

The source of distortions in covered banking is the limit imposed on the provision of liquidity insurance. The first best level of liquidity insurance violates the run preventive constraint; therefore, a covered bank will provide strictly lower early payoffs than in the first best. The incentive to increase early consumption towards the first-best level implies that the run-preventive constraint binds for all levels of wealth ($c_E = w - k + \gamma h(k)$). This limit on early consumption forces the bank to provide a constant level of liquidity insurance over regions A, B and C ($c_E = \gamma c_L$) below the efficient level. Lower liquidity insurance frees resources to provide higher late consumption either through reducing liquidation or increasing capital investment.[30]

[28] See property 6 in Appendix B for a formal proof.

[29] $k_C^c(w)$ and $k_D^c(w)$ are implicitly defined by the expressions of liquidity insurance and excess liquidity The thresholds that define regions C and D are: $\widetilde{w}^c = \underline{k}\left(1 + \frac{\pi}{\beta(1-\pi)}\right)$, and $\hat{w}^c = \widehat{k}^c\left(1 + \frac{\gamma\pi}{\beta(1-\pi)}h'\left(\widehat{k}^c\right)\right)$, where: $h'\left(\widehat{k}^c\right) = \frac{\pi+(1-\pi)\gamma^\sigma}{\pi\gamma+(1-\pi\gamma)\gamma^\sigma}$. The details of the derivation of the optimal covered contract are provided in the supplemental appendix.

[30] Over regions A and B, investment is determined by pure technological considerations ($k = w$ and $k = \underline{k}$). Lower liquidity insurance implies a smaller liquidation of the long asset $\lambda^c(w) < \lambda^u(w)$ and a faster convergence to region C ($\widetilde{w}^c < \widetilde{w}^u$). There is an ineffcient provision of liquidity insurance as increases in capital over region C are not accompanied by increases in liquidity insurance. Over region C and the first part of D, the bank “overinvests” in capital to maintain a covered contract. In the second part of region D, there is "underinvestment" in capital relative

Under covered banking, crisis prevention is obtained by restricting the provision of liquidity insurance and by choosing an investment portfolio that ensures that depositors' claims will always be satisfied. In the previous literature, a requirement of excessive liquid reserves can attain this objective. However, when returns are endogenous, it is not necessarily the case. We find that, except for rich economies, it is more efficient to reduce the promises to early consumers rather than to hold more liquid assets. This reduction of liquidity insurance allows the bank to allocate *more* resources on long-term projects, with positive consequences for economic growth.

The marginal cost in terms of liquidity insurance imposed by the binding run preventive constraint $c_E = w - k + \gamma h(k)$ can be analyzed using the shadow value of this constraint relative to the shadow value of the resource constraint for late consumption, $(1-\pi)c_L = w - k + \lambda\gamma h(k) + (1-\lambda)h(k) - \pi c_E$. This marginal cost of run prevention corresponds to the distortion of covered banking with respect to the first best solution. In Appendix B.2, we show how this measure evolves with the level of wealth. Over regions A and B, the marginal cost of run prevention is constant since both the first best and the covered solution provide a constant level of liquidity insurance. Over region C, this marginal cost is increasing since run prevention forces keep liquidity insurance constant, which would have increased under the first best solution. Finally, over region D, the marginal cost of run prevention decreases. By holding excess liquidity, the bank can remain run-proof while gradually relaxing the constraint on liquidity insurance. For large levels of wealth, the marginal cost of run prevention eventually vanishes as the level of liquidity insurance converges asymptotically to the first best.[31]

3.1.4 Exposed Banking ($\boldsymbol{\eta} = \mathbf{q}$).

Proposition 3.3 *The optimal exposed banking contract for high risk aversion ($\sigma > 1$) is characterized by the following conditions.*

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to the first-best, as "excess liquidity" ($i > 0$) becomes a more efficient way to restrict liquidity insurance. The use of excess liquidity before fully exhausting the return on the long asset ($h'(k) < 1$) is a technological inefficiency of covered banking. Over region D, the bank can maintain a covered contract and increase liquidity insurance, reducing the distortion generated by the run-preventing constraint.

[31] Since a covered bank is protected against inefficient liquidation, the level of capital investment will also converge asymptotically to the first best level.

	Wealth: w	Capital: k^u	Liquidation: λ^u	Excess Liquidity: i^u	Liquidity Insurance: $\left(\frac{c_E}{c_L}\right)^\sigma$
A	$0 < w \leq \underline{k}$	w	λ^*	0	γ
B	$\underline{k} \leq w \leq \tilde{w}^u$	$\underline{k}$	$\lambda^* - (1-\lambda^*)\,\beta\frac{w-\underline{k}}{\underline{k}}$	0	γ
C	$\tilde{w}^u \leq w \leq \hat{w}^e$	$k^e_C(w)$	0	0	$\left[\begin{array}{c} h'(k^e) \\ -\frac{q}{1-q}\left(1-\gamma h'(k^e)\right)\frac{u'(c_E)}{u'(c_L)} \end{array}\right]^{-1}$
D	$w \geq \hat{w}^e$	$k^e_D(w)$	0	$(1-\pi)(w-k^e) - \pi h(k^e)$	1

where $k^e_C(w)$ and $k^e_D(w)$ are strictly increasing in the level of wealth.[32] Figure 4b illustrates the optimal choice of capital and liquidity insurance for a simulation of the economy.

Regions A and B of the exposed contract are identical to the first-best contract. Over these regions the level of investment is determined by technological efficiency, and it is optimal to provide the first-best level of liquidity insurance, since a reduction of liquidity insurance helps only if it makes the contract run proof (covered banking); otherwise, crises are still possible.[33] Since there are no distortions in the exposed contract over these regions, the cost associated with the exposed banking solution is just the expected cost of a crisis.

Exposed banking introduces an important new element. Having crises with positive probability generates aggregate uncertainty in the payoffs for both types of consumers. The bank will have incentives to smooth consumption over realizations of the aggregate state, that is to mitigate the expected cost of crisis. *Crisis mitigation* is achieved by increasing the liquidation value of the bank's portfolio in the early sub-period. Since the early value of the portfolio increases with investment in the storage technology, the bank will invest less capital than the optimal risk sharing over regions C and D.

[32] $k^e_C(w)$ is implicitly defined by the expressions of liquidity insurance and excess liquidity. $k^e_D(w)$ is implicitly defined by:

$$(1-q)\left(h'(k)-1\right)(w-k+\gamma h(k))^\sigma = q\,(w-k+h(k))^\sigma\left(1-\gamma h'(k)\right)$$

The threshold $\hat{w}^e$ is given by:

$$\hat{w}^e = \hat{k}^e\left(1+\frac{\pi h'\left(\hat{k}^e\right)}{\beta(1-\pi)}\right) \quad where\ : h'\left(\hat{k}^e\right) = \frac{q+(1-q)(\pi+(1-\pi)\gamma)^\sigma}{\gamma q+(1-q)(\pi+(1-\pi)\gamma)^\sigma}$$

The details of the derivation of the optimal exposed contract are provided in the supplemental appendix.

[33] Even though an exposed bank replicates the first best contract over these regions, it does not attain the first best solution because of the expected costs of crisis.

There is no conflict for the exposed bank between increasing liquidity insurance and reducing the cost of crisis. By adding extra liquidity the bank can promise higher early returns and, at the same time, increase its bankruptcy value. Over region C, the bank provides an inefficiently high level liquidity insurance. It also starts to provide full liquidity insurance for a lower level of wealth than under the first best contract($\hat{w}^e < \hat{w}^u$). Over regions C and D, the main distorsion of exposed banking contract is a higher marginal cost of investing in capital, which reflects the cost of crisis mitigation.[34] However, as wealth increases, the opportunity cost of holding more liquidity to mitigate the cost of crisis decreases.

3.1.5 The Optimal Banking System

In this section, we characterize the optimal risk-sharing solution when panic crises are possible as the choice between the optimal "covered" and "exposed" contracts. For any given level of wealth, the financial intermediary will choose the contract that maximizes expected utility. The bank's decision reflects the tension between crisis prevention and crisis mitigation**.** The financial intermediary chooses $\eta = \arg\max \{V(\eta, w)\}$, where $V(\eta, w) = Max\{V^e(q, w), V^c(w)\}$.

Since the distortions generated by the contracts vary with the level of wealth, the optimal choice between the contracts will depend on wealth and on the probabilty of a bad realization of the sunspot. The expected utility of covered banking ($V^c(w)$) is invariant to q, while the expected utility of the exposed contract ($V^e(q, w)$) is strictly decreasing in q. The choice between the two contracts will be determined by a wealth-dependent, cut-off probability $q^*(w)$. This threshold probability is defined in the following proposition.

Proposition 3.4 ***The Optimal Banking System***

For any level of wealth if $\sigma > 1$, and for $w > w_{rp}$ if $\sigma \leq 1$, there exists a unique cut-off probability $q^* \in (0, \pi]$ such that:

$$q > q^*(w) \Leftrightarrow \textit{a covered banking system is optimal}$$

$$q < q^*(w) \Leftrightarrow \textit{an exposed banking system is optimal}$$

where $q^*(w)$ is a continuous function defined by:

$$V^e(q^*(w), w) = V^c(w)$$

[34] In the first best solution the marginal cost of investing in capital is $u'(c_E)$, and the marginal benefit $u'(c_L)h'(k)$. In an exposed banking system the marginal cost of investing in capital becomes $(1-q)u'(c_E) + qu'(c_R)$, and the marginal benefit $[(1-q)u'(c_L) + q\gamma u'(c_R)]h'(k)$.

Proof. See Appendix B ■

Over regions A and B and exposed banking system replicates the first-best contract, and the only cost is the expected cost of a run. This cost increases with q and, therefore, the expected utility is decreasing in q.

Over regions C and D, a positive probability of a run q increases the liquidation risk, reducing the expected marginal return of capital and investment. Lower capital investment for a given level of wealth has two effects on welfare: a positive effect because it increases liquidity insurance and a negative effect because it reduces the returns for late consumption. The overall effect of the increase in q is negative because the bank is increasing the expected payoff in case of a run at the cost of reducing it when there is no run, exacerbating the distortion in the non-run case.[35]

In Appendix B, we show that if the probability of the sunspot is higher than the probability of the idiosyncratic liquidity shock ($q > \pi$), autarky dominates the exposed banking solution. Since covered banking weakly dominates the autarkic outcome, the cutoff probability $q^*(w)$ must be strictly lower than π.

The cutoff probability determines the bank's optimal choice of contract for any given level of wealth. It is, however, useful to invert the problem and find, for a given probability of the sunspot, how the decision between the two contracts changes with the level of wealth. This analysis sheds light on how the exposure to banking crises varies over the development path, and it shows the cross-sectional distribution of crisis risk in equilibrium for countries with different levels of wealth.

Proposition 3.5 ***Optimal Banking and the Level of Wealth.***

There exist two cutoff probabilities $q_0,\ q_1$ *(*$0 < q_0 < q_1 < \pi$*) such that:*

(i) high probability of a run: *if* $q > q_1$, *a covered banking system is the optimal for all levels of wealth*

(ii) intermediate probability of a run: *if* $q_0 < q < q_1$, *there exist two levels of wealth* $w_l < w_h$ *such that an exposed banking system is optimal for middle income economies* $(w_l < w < w_h)$ *and a covered banking system is optimal for poor and rich economies* $(w \in \mathbb{R}^+ - [w_l, w_h])$

[35] Using the envelope theorem we can see that:

$$\frac{dV^e(q,w)}{dq} = -\left[\pi u(c_E) + (1-\pi)u(c_L)\right] + u(c_R) < 0.$$

(iii) low probability of a run: *if* $q < q_0$, *there exist one level of wealth* w_h *such that an exposed banking system is optimal, except for rich economies* $(w > w_h)$

where:

$$q_0 = q_o^* = \frac{\left[\pi+(1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} - [\pi+(1-\pi)\gamma^{\sigma-1}]}{\left[\pi+(1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} - 1}$$

$$q_1 = Max\left\{q^*(w)\right\} < \pi$$

$$\frac{\delta w_l}{\delta q} > 0; \frac{\delta w_h}{\delta q} < 0 \text{ } and \lim_{q \to 0} w_h = 0$$

Proof. See Appendix B ■

Figure 5 illustrates the characterization of the optimal solution in Proposition 3.5. For any probability of the pessimistic state q, it shows the upper and lower wealth thresholds (w_h and w_l) that define the switch between the two contracts.

For poor economies, the cost of covered banking is the low liquidity insurance provided by the intermediary; however, the cost is partially compensated because lower liquidation increases late consumption. On the other hand, since exposed banking replicates the unconstrained contract, the cost of exposed banking is the cost of a run. Therefore, poor economies will prefer a covered contract when the probability of the pessimistic state is high enough ($q > q_0$) because a high probability of crisis implies a higher welfare loss than the sacrifice of liquidity insurance under covered banking.

The underinsurance distortion of covered banking becomes more pervasive for higher levels of wealth (region C) since liquidity insurance is kept constant even when returns to the long-term technology are decreasing.[36] On the other hand, an exposed contract does increase liquidity insurance and mitigates the adverse effects of crises at a decreasing opportunity cost, partially offsetting the losses associated with bank runs. Therefore, for intermediate levels of wealth, the exposed contract may prevail over the covered contract (if $q < q_1$). However, there is always a sufficiently high probability q that can make the exposed banking suboptimal.

For richer economies (region D), the distortions associated with covered banking are reduced. A bank can remain covered by holding excess liquidity while improving liquidity insurance. Furthermore, the level of investment in capital under covered banking tends asymptotically towards the first best maximum level ($\overline{k}$). In contrast, an exposed bank always faces an uninsurable crisis risk

[36] As we discuss in section 3.1.3, the marginal cost of satisfying the run preventing constraint is constant over Region A and B, increasing over Region B and finally decreasing over Region C. This result is formally derived in Appendix B.2

that prevents capital investment to achieve the maximum efficient level.[37] Hence, there is always a high enough level of wealth beyond which covered banking is the optimal contract.

4 Growth and the Output Costs of Liquidity Crises

4.1 Growth and Convergence under Optimal Banking

We characterize the dynamics of wealth implied by the optimal banking solution for high risk aversion ($\sigma > 1$).[38] We assume an initial generation endowed with $w_0 > 0$. When the optimal contract is $j = \{c, e\}$, the dynamics of wealth can be represented as:

$$w_t = \begin{cases} F^j(w_{t-1}) = & (1-\beta)\left[\lambda(w_{t-1})\gamma + 1 - \lambda(w_{t-1})\right] f(k(w_{t-1})) & \text{with probability } 1-\eta \\ F^{run}(w_{t-1}) = & (1-\beta)\gamma f(k(w_{t-1})) & \text{with probability } \eta \end{cases} \tag{13}$$

$k_t = k^j(w_{t-1})$: optimal capital choice

$\lambda_t = \lambda^j(w_{t-1})$: optimal liquidation

$$\eta = \begin{cases} 0 & \text{if } j = c \\ q & \text{if } j = e \end{cases}$$

When the optimal banking solution is a *covered banking system ($j = c$)*, the dynamics of wealth are *deterministic*. By contrast, when the optimal banking solution is an *exposed banking system ($j = e$)*, the dynamics of wealth are *stochastic*. When an exposed bank experiences a run, full liquidation of the bank's porfolio will reduce wealth and investment possibilities of the following generation.

In the appendix, we show that $F^c(w)$, $F^e(w)$, and $F^{run}(w)$ have unique fixed points $\bar{w}^c, \bar{w}^e$ and $\underline{w}^e$ respectively. The convergence properties of this economy are summarized by the following proposition:

Proposition 4.1 *For any initial wealth $w_0 > 0$,*

[37] In the limit for infinitely large wealth k^c attains $\overline{k}$, while k^e attains an upper bound given by:

$$h'\left(k^e_{\max}\right) = \frac{1}{q\gamma + 1 - q} \Leftrightarrow k^e_{\max} < \overline{k}$$

[38] Early and late consumption (c_E and c_L) are monotonically increasing in wealth; therefore, as in the case of autarky, their dynamics follow the dynamics of wealth and the level of liquidity insurance implied by the optimal contract.

(i) *if* $q > q_1$ *or* $w^h \leq \bar{w}^c$ *the economy with an optimal banking system converges to a long run steady state* ($F^c(\bar{w}^c) = \bar{w}^c > 0$) *characterized by a covered banking system.*

ii) *otherwise, the economy in the long run displays stochastic fluctuations within the range* $\left(\underline{w}^e, \bar{w}^e\right)$ *under an exposed banking system.*

where q_1 *and* w^h *are defined in Proposition 3.5.*

Proof. See Appendix B ■

Figure 6 illustrates the dynamics for a simulation of the economy. It presents the unique dynamic paths ($F(w_{t-1})$) for autarky, covered banking, and the unconstrained problem. In contrast, the stochastic growth dynamics for exposed banking is represented by two paths: $F^e(w_{t-1})$ if there is no run, and $F^{run}(w_{t-1})$ otherwise. The dynamics of the optimal banking solution are underlined. The steady state is determined by the intersection of the optimal path with the 45 degree line.

The simulation used in Figure 6 presents the case of an economy with an intermediate probability of the sunspot ($q_0 < q < q_1$).[39] Covered banking is the optimal contract for both low and high incomes and attains a covered-banking steady state. Starting with an initial low level of wealth, such an economy experiences the fast growth associated with covered banking and then switches to an exposed contract, entering the region where crises happen with positive probability. Eventually, the economy will converge to a long-run, financially-safe steady state. The speed of convergence will depend on the realization of the sunspot. If the economy receives good draws it will "*escape*" rapidly to a run-proof region. If the economy experiences bad draws, it will experience multiple crises, and yet, it remains optimal to take on the risk associated with an exposed banking system.

The optimality of covered banking for high levels of wealth is similar to the result of Acemoglu and Zilibotti [1997]. In their model growth and crises will depend on "*luck*" until the economy gets rich enough to afford full insurance through broader risk diversification. In our model, the economy is financially fragile and vulnerable to bank runs until it becomes rich enough to afford the cost of a full self-insurance against the risk of liquidity crises.

4.2 The Growth Process: Financial intermediation vs. Financial autarky

In this section, we compare the growth processes under financial intermediation and under autarky.[40] We concentrate on growth under a covered banking system leaving the analysis of output

[39] The parameters used in the simulation are presented in Appendix D.

[40] For simplicity, we restrict our attention to the most interesting case when $\sigma > 1$.

losses caused by banking crises to the next section. The relative growth rates of the two financial arrangements can be analyzed using the ratio of wages for the following generation:

$$\frac{F^a(w)}{F^c(w)} = \frac{1-\pi(1-\gamma)}{1-\lambda(w)(1-\gamma)} \left(\frac{k^a(w)}{k^c(w)}\right)^\beta \tag{14}$$

where the indexes $\{a, c\}$ stand for financial autarky and covered banking system.

Equation (14) can be written in log differences as:

$$g^a(w) - g^c(w) \approx \underbrace{\ln(1-\pi(1-\gamma)) - \ln(1-\lambda(w)(1-\gamma))}_{\text{Liquidation Effect}} + \beta \underbrace{[\ln k^a(w) - \ln k^c(w)]}_{\text{Investment Effect}} \tag{15}$$

The growth differential between financial autarky and financial intermediation depends on the combination of a liquidation effect, which is determined by differences in the level of liquidation ($\lambda(w)$ vs π), and an investment effect, which is determined by differences in capital choice. In terms of growth accounting, the first effect reflects the difference in *total factor productivity* (TFP) and the second effect the difference in *capital investment.*

Under autarky, self-insurance imposes a constant aggregate liquidation equal to π for all levels of wealth. Hence, the level of TFP is constant and the growth process is entirely neo-classical. In contrast, under financial intermediation, the economy evolves successively through an *endogenous growth regime* and a *neo-classical growth regime.* During the endogenous phase (region B), the level of capital investment is kept constant and the bank increases investment in the short-term technology in order to reduce the liquidation of long-term projects. This reduction in costly liquidation endogenously increases the level of total factor productivity. As capital is constant, the endogenous change in TFP is the engine of growth. This growth regime ends when the level of liquidation ($\lambda(w)$) reaches zero. In the subsequent neo-classical phase (region C and D), growth is driven by capital investment and the economy under covered banking converges to a long run steady state.

The Liquidation Effect

Under financial intermediation, whenever the marginal return of the short technolgy exceeds the early liquidation marginal return of the long technology, the bank sets liquidation to zero. This feature represents a *technological advantage* of banking, that is, its ability to avoid inefficient liquidation by pooling liquidity risk. Hence, financial intermediation is associated with a higher long run level of total factor productivity.[41]

[41] The long run levels of TFP under autarky and covered banking are respectively $A(\lambda\gamma\pi + 1 - \pi) < 1$ and A.

The Investment Effect

For some low levels of wealth ($w \in [\underline{k}, w^*]$), autarkic agents overinvest in capital as precautionary savings, while it would be efficient to start investing a fraction of wealth in the short technology and to reduce liquidation. As a result of inefficient overinvesment, growth can be higher under autarky than under financial intermediation for low levels of wealth. Nevertheless, the cost of inefficient liquidation under autarky limits capital investment for larger levels of wealth, and investment in capital and growth eventually become higher under financial intermediation.

Under financial intermediation, the reduction of liquidation results in a permanent increase in TFP that induces higher investment and higher growth during the subsequent neo-classical growth phase.[42] Hence, the long run level of capital and wealth are higher under financial intermediation than under autarky.

Figure 6 also illustrates the stage at which the development of a banking system starts to have crucial long-run effects. When the economy has enough resources to keep an increasing number of long term projects until full maturity, financial intermediation has an increasing contribution to growth. This result replicates the empirical importance of financial intermediation for the growth perspectives of middle-income or emerging economies.

4.3 Liquidity crises and output losses

An exposed bank is vulnerable to panic runs, and runs impose a cost on the current and future generations. The ultimate cost of a financial crisis is the reduction in welfare it imposes on consumers of the current, and any subsequent generation that may bear the costs. The output foregone when there is a crisis is another possible indicator of its cost. This measure synthesizes both the loss of consumption of the current generation, and the reduction in investment (or wealth) of the next generation. The output under exposed banking is:

$$\begin{aligned} y &= w - k^e + \left(1 - \lambda^e \left(1 - \gamma\right)\right) f\left(k^e\right) && \text{if there is no run} \\ y_r &= w - k^e + \gamma f\left(k^e\right) && \text{if there is a bank run} \end{aligned}$$

We define the relative output loss by:

$$L_Y = \frac{y - y_R}{y} = \frac{\left(1 - \lambda^e\right)\left(1 - \gamma\right)}{\beta \frac{(w - k^e)}{h(k^e)} + \left(1 - \lambda^e\left(1 - \gamma\right)\right)} \tag{16}$$

Output foregone in case of a run is linked to the liquidity of the banking portfolio. The more liquid the portfolio, the lower the output cost in case of a crisis, because there is less inefficient

[42] A similar feature can be found in the model of Matsuyama (1999) where an economy grows through succesive phases of endogenous technical change and capital accumulation.

liquidation of long-term projects. The bank provides liquidity by investing in the short technology $(w-k)$ and by liquidating a proportion λ of long-term projects. The following table presents the relative output loss for the different regions of exposed banking:

Region	$\mathbf{w-k}$	$\boldsymbol{\lambda}(\mathbf{w})$	$\mathbf{L_Y}$	
A	0	λ^*	$\frac{(1-\lambda^*)(1-\gamma)}{1-\lambda^*(1-\gamma)}$	constant
B	increasing	decreasing	$\frac{\beta(w-\underline{k})+\underline{k}}{\beta(w-\underline{k})+\underline{k}+\frac{\gamma}{(1-\lambda^*)(1-\gamma)}\left[\beta^2(w-\underline{k})+\underline{k}\right]}$	increasing
C	increasing	0	$\frac{(1-\gamma)}{1+\beta\frac{w-k}{h(k)}}$	decreasing
D	increasing	0	$\frac{(1-\gamma)}{1+\beta\frac{w-k}{h(k)}}$	decreasing

The relative output loss $L_{\mathbf{Y}}$ exhibits a humped shape. Over region A, a constant fraction of output is liquidated in normal times. Hence, the relative output loss is constant. Over region B, there are two opposite effects: first, an exposed bank starts investing in the liquid asset, which reduces the relative output lost; second, it decreases the optimal level of liquidation, increasing the relative output lost. The latter effect dominates, and increases of wealth over this region increases the loss in case of a run. A crisis can be interpreted as a drastic reduction in total factor productivity. Since financial intermediation over region B is increasing total factor productivity by reducing liquidation of the long-term technology, the relative output loss increases over this region. Once an exposed bank stops liquidating the long technology, any subsequent increase in wealth will be accompanied by an increase in investment in the liquid asset, thus reducing the output loss in case of a run.[43]

Figure 7 depicts the potential output loss for an exposed banking system under different probabilities of the sunspot q. Over regions A and B, an increase in q does not induce banks to hold more liquid assets in order to mitigate the costs of crisis. The only way for a bank to avoid the consequences of runs is to be covered. In contrast, over regions C and D, an exposed bank reduces the output costs of crisis through holding a more liquid portfolio. This incentive for crisis mitigation increases with the probability of a run. The humped shape of the output loss matches the empirical evidence: crises in middle-income economies have higher costs than poor and rich economies.

[43] Except for region B, there is a negative relationship between the output loss and liquidity insurance, since early consumption is increased using liquid assets.

5 Conclusion

In this paper, we developed an integrated framework to analyze the relationships between financial intermediation, financial fragility and growth. This framework is capable of replicating both the observed relationship between financial development and economic growth, and between the level of economic development and the recurrence and depth of financial crises.

The results can be summarized as follows. Poor economies have too much to lose in a banking crisis and have incentives to sacrifice liquidity insurance for crisis protection; middle income-economies may choose to remain vulnerable to crises in exchange for higher liquidity insurance and returns; and finally, rich economies bear a smaller cost for being fully protected against crises and avoiding any liquidation of long-term projects. By choosing to stay vulnerable to runs, middle-income economies optimally choose to mitigate rather than to prevent banking crises. As they get richer, they may eventually converge to a long-run, financially-safe steady state. As a result, the macroeconomic volatility associated with the risk of crises may be only a temporary phenomemon on the road to development.

Consistent with the data, we find that the development of the banking system in middle-income economies is associated with both higher growth and a higher risk of banking crises. The model is also consistent with the empirical evidence that the output costs of banking crises are more severe for middle-income economies than for rich economies.

Our model is designed to be simple enough to analyze the endogenous evolution of financial intermediaries in a tractable way. A natural extension would be to introduce banking crises driven by aggegate shocks. As long as the trade-off between liquidity and returns continue to vary with the level of wealth, this extended framework should deliver similar results. Another possible direction would be to consider the role of inter-generational arrangements for the stability and the efficiency of the banking system. Finally, investigating how some other functions of banks - e.g. monitoring loans, screening investment projects - may evolve with the level of economic development seems to be a promising avenue for future research.

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Appendix

A Financial Autarky

Under financial autarky, young agents make their investment decision between storing goods and investing in capital on their own.

A.1 The optimal individual investment decision

At the end of their first period, for any given level of wealth $w > 0$, a typical agent of generation t chooses investment in the long technology k to maximize:

$$\pi u\left(c_E\right) + (1-\pi)u\left(c_L\right) \tag{17}$$

$$\text{subject to } \; 0 \leq k \leq w \tag{18}$$

where $c_E = w - k + \gamma h(k)$, $c_L = w - k + h(k)$, and the difference between wealth and capital $(w-k)$ represents investment in the storage technology.

The optimal solution for members of any given generation under financial autarky is characterized by:

$$(i) \text{ if } w \leq w^* \text{then} \begin{cases} k_{opt}(w) = w \\ c_E = \gamma h(w) \\ c_L = h(w) \end{cases} (\textbf{corner solution})$$

$$(ii) \text{ if } w > w^* \text{ then } \begin{cases} 0 < k_{opt}(w) < w \\ c_E = w - k_{opt} + \gamma h(k_{opt}) \\ c_L = w - k_{opt} + h(k_{opt}) \end{cases} (\textbf{interior solution})$$

where $k_{opt}(w)$ in (ii) is defined by $\frac{u'(c_E)}{u'(c_L)} = \frac{(1-\pi)(h'(k_{opt})-1)}{\pi(1-\gamma h'(k_{opt}))}$ and $w^* \in (\underline{k}, \overline{k})$ is implicitly defined by $\frac{u'(\gamma h(w^*))}{u'(h(w^*))} = \frac{(1-\pi)(h'(w^*)-1)}{\pi(1-\gamma h'(w^*))}$.

The optimal solution under autarky is inefficient. In poor economies self insured agents invest, as precautionary savings, their full wealth in capital beyond the point where it is efficient to do so ($w^* > \underline{k}$, $\gamma h'(w^*) < 1$).

For $w > w^*$, there is inefficient aggregate liquidation of the long technology, and for any level of wealth, investment in capital is bounded above by $k_{\max}$ ($h'(k_{\max}) = \frac{1}{\pi\gamma+(1-\pi)} > 1$)

A.2 Growth under Financial Autarky

Since capital fully depreciates after it is used, the connection between the individual problem and the dynamics of the intertemporal model is given by wages of the next generation:

$$w_t = F^a(w_{t-1}) = (1-\beta)(\pi\gamma + 1 - \pi)f(k(w_{t-1})) \tag{19}$$
$$k_t = k(w_{t-1}) = k_{opt}(w_{t-1})$$

The following proposition characterizes the dynamics of this economy:

Proposition A.1 *The economy converges to a unique stable steady state* $\bar{w}^a > 0$ *and* $k(\bar{w}^a)$*. The steady state is defined by* $F^b(\bar{w}^a) = \bar{w}^a$

Proof. See Supplemental Appendix ■

Figure 6 includes the dynamics of wealth under autarky. Beyond the threshold w^*, the rate of growth decreases rapidly, since overinvestment in the previous region has already exhausted the marginal returns on capital. A constant level of liquidation π, due to self insurance, becomes more and more costly in terms of growth.

B The optimal banking system

B.1 Optimal banking system and unconstrained risk-sharing (proof of 3.1)

Lemma B.1 *The first best solution satisfies* $c_E \leq c_R$*, if and only if* $\sigma \leq 1$ *and* $w \leq w_{rp}$ *where* $w_{rp} = k_{rp}(1 + \frac{\pi\gamma}{(1-\pi)\beta\gamma^\sigma})$ *and* $h'\left(k_{rp}\right) = \frac{1}{\gamma^\sigma}$

The condition $c_E \leq c_R$ in the four regions of the unconstrained solution requires:

Region A: $\frac{c_R}{c_E} \geq 1 \Leftrightarrow \lambda^* \leq \pi$.

Region B: $\frac{c_R}{c_E} \geq 1 \Leftrightarrow (\lambda^* - \pi)\left(\beta\left(w - \underline{k}\right) + \underline{k}\right) \leq 0 \Leftrightarrow \lambda^* \leq \pi$.

Region C: $\frac{c_R}{c_E} \geq 1 \Leftrightarrow \frac{c_E}{c_L} \leq \gamma$

Region D: $\frac{c_R}{c_E} \geq 1 \Leftrightarrow \gamma \geq 1$, (which is impossible since $\gamma < 1$).

(i) The first best solution does not satisfy $c_E \leq c_R$ if $\sigma > 1$

Region A and B: $\sigma > 1 \Rightarrow \lambda^* > \pi$.

Region C: Optimality requires $1 \leq \frac{u'(c_E)}{u'(c_L)} = h'(k) \leq \frac{1}{\gamma} \Rightarrow \frac{c_E}{c_L} \geq \gamma^{\frac{1}{\sigma}}$

since $\sigma > 1 \Rightarrow \gamma^{\frac{1}{\sigma}} > \gamma$ which contradicts $\frac{c_E}{c_L} \leq \gamma$.

(ii) The optimal risk sharing solution satisfies $c_E \leq c_R$ if $\sigma \leq 1$ and $w \leq w_{rpc}$

Region A and B: $\sigma \leq 1 \Rightarrow \lambda^* < \pi$, optimal risk sharing satisfies $c_E \leq c_R$.

Region C: Optimality requires: $\gamma^{\frac{1}{\sigma}} \leq \frac{c_E}{c_L} \leq 1$, and $\frac{c_E}{c_L} = h'(k)^{-\frac{1}{\sigma}}$. Since $\sigma \leq 1 \Rightarrow \gamma^{\frac{1}{\sigma}} < \gamma < 1$ and $k(w)$ is strictly increasing in w, then there exists a unique level of wealth (w_{rp}), and a unique capital level (k_{rp}), such that the condition $\frac{c_E}{c_L} \leq \gamma$ is violated. k_{rp} is defined by $h'(k(w_{rp})) = \left(\frac{1}{\gamma}\right)^{\sigma}$

The proof of 3.1 follows three steps:

(i) Any feasible contract of the general banking problem $\left(P^0\right)$ is a feasible allocation for the UORS program (P^u). The set of feasible allocations in P^0 is a subset of the set of feasible allocations for P^u. Therefore the unconstrained optimal risk sharing solution (UORS) dominates the optimal exposed banking solution.

(ii) If for a given level of w, the exposed banking solution is vulnerable to runs $c_E^e > c_R^e$ then

$$\begin{aligned}
(1-q)\left[\pi u(c_E^e) + (1-\pi)u(c_L^e)\right] + qu(c_R^e) &< (1-q)\left(\pi u(c_E^e) + (1-\pi)u(c_L^e)\right) + qu\left(c_E^e\right) \\
&\leq \pi u(c_E^e) + (1-\pi)u(c_L^e) \quad \text{since } c_E^e \leq c_L^e \\
&\leq \pi u(c_E^u) + (1-\pi)u(c_L^u)
\end{aligned}$$

Thus, $V^e(q,w) < V^u(q,w)$ (i.e. the exposed solution is strictly dominates by the first best solution)

(iii)If for a given level of w in the UORS solution $c_E^u(w) < c_R^u(w)$, the run preventive constraint is not binding in the covered banking program, then the first best solution and the covered banking solution coincide. By lemma 1, this is the case if and only if $\sigma \leq 1$ and $w \leq w_{rp}$.

B.2 Covered Banking: the marginal cost of the run preventive constraint.

The cost on liquidity insurance imposed by the run preventive constraint can be defined as the multiplier of the run preventive constraint normalized by the total resource constraint. If μ_3 is the Lagrange multiplier of the run preventive constraint ($c_E \leq w - k + \gamma h(k) \equiv c_R$), and μ_2 is Lagrange multiplier of the second sub-period budget constraint ($(1-\pi)c_L \leq w - k + \lambda\gamma h(k) + (1-\lambda)h(k) - \pi c_E$). The marginal cost on liquidity insurance of the run preventive constraint is measured as a marginal rate of substitution $\left(\frac{\mu_3}{\mu_2}\right)$, defined by the marginal cost of in terms of early consumption lost by keeping the constraint (μ_3) relative to the marginal gain in terms of late consumption (μ_2). As we prove in the supplemental appendix, the evolution of this cost in the process of development exhibits an inverted U-shape. The following table indicates the values taken by the marginal cost of the run preventive constraint:

Region	$\frac{\mu_3}{\mu_2}$	$\frac{d\frac{\mu_3}{\mu_2}}{dw}$
A	$\pi\left(\frac{1}{\gamma^\sigma}-\frac{1}{\gamma}\right)$	0
B	$\pi\left(\frac{1}{\gamma^\sigma}-\frac{1}{\gamma}\right)$	0
C	$\pi\frac{\left(\frac{1}{\gamma^\sigma}-h'(k)\right)}{1-\pi(1-\gamma h'(k))}$	$\frac{-h''(k)}{1-\pi(1-\gamma h'(k))}\left(1+\pi\gamma\frac{\mu_3}{\mu_2}\right)\frac{dk}{dw}>0$
D	$\frac{h'(k)-1}{1-\gamma h'(k)}$	$\frac{h''(k)}{1-\gamma h'(k)}\left(1+\gamma\frac{\mu_3}{\mu_2}\right)\frac{dk}{dw}<0$

Since $\lim_{w\to\infty} k^c(w)=\bar{k}\Rightarrow \lim_{w\to\infty}\frac{\mu_3}{\mu_2}=0.$

B.3 Properties of the Value Functions.

P1: $V^e(w,q)$ and $V^c(w)$ are continuous, differentiable, strictly increasing and strictly concave functions in w and satisfy Inada Conditions.

P2: $V^e(w,q)$ is a continuous, differentiable and strictly decreasing function in q.

P3: $V^c(w)$ is invariant in q

P4: $V^c(w)<V^u(w)$ and $\lim_{w\to\infty}\frac{V^c(w)}{V^u(w)}=1$

P5: $q>0:V^e(w,q)<V^u(w)$; $\lim_{w\to\infty}\frac{V^e(w,q)}{V^u(w)}<1$ and $\lim_{q\to 0}\frac{V^e(w,q)}{V^u(w)}=1$; and $q=0\Rightarrow V^e(w,0)=V^u(w)$:

P6 Covered Banking weakly dominates autarky ($V^c(w)\geq V^a(w)$) and strictly dominates autarky for $w>\underline{k}$

- By replicating the autarkic solution ($\lambda=\pi, k=k^a(w)$), a bank is covered⇒covered banking weakly dominates autarky
- The solution for the optimal covered bank is unique. Therefore, except when the autarkic and covered banking solution are identical ($w\leq\underline{k}$), the optimal covered banking solution strictly dominates autarky.

B.4 The optimal banking system [proof of proposition (3.4)]

First, we prove existence by showing that for extreme values of q (0 and π) the choice of the optimal contract differs. Uniqueness comes from a single crossing property, given the properties of the value functions.

- for $q=0:V^e(w,q)=V^u(w)>V^c(w)$

- for $q = \pi$: covered banking weakly dominates exposed banking. First, we show that autarky weakly dominates exposed banking.

Under autarky, the objective function is:

$$V^a(w) = \max\{\pi u(w - k + \gamma h(k)) + (1-\pi)u(w - k + h(k))\}$$

Under exposed banking, the objective function is:

$$\begin{aligned} V^e(q = \pi, w) &= \max\{(1-\pi)[\pi u(c_E) + (1-\pi)u(c_L)] + \pi u(w - k + \gamma h(k))\} \\ \text{with} &: \quad \pi c_E + (1-\pi)c_L \leq w - k + h(k) \end{aligned}$$

⇒By Jensen inequality:

$$[\pi u(c_E) + (1-\pi)u(c_L)] \leq u(w - k + h(k))$$

the optimal solutions for autarky and exposed banking when $q = \pi$, imply $k^e(w) \leq k^a(w)$ for any $w < \infty$, and

$$\lim_{w\to\infty} k^e(w)\Big|_{q=\pi} = \lim_{w\to\infty} k^a(w) = \frac{1}{\pi\gamma + 1 - \pi}$$

then:

$$V^e(w) \leq V^a(w)$$

Using $P6$:

$$V^e(w, \pi) \leq V^a(w) \leq V^c(w)$$

for $q > \pi$ by $P2$ and $P6 : V^e(w) < V^a(w) \leq V^e(w)$

By $P2$ and $P3$ the cutoff probability $q^*(w)$ is unique, therefore:

$$\begin{aligned} q &< q^*(w) : V^e > V^c \\ q &> q^*(w) : V^e < V^c \\ q &= q^*(w) : V^e = V^c \end{aligned}$$

- $q^*(w)$ is implicitly defined by:

$$V^e(w, q^*) = V^c(w)$$

then as $V^e(w, q^*)$ and $V^c(w)$ are continuous in w and $V^e(w, q)$ is continuous in q, $q^*(w)$ is continuous in w

B.5 Optimal Banking and the level of wealth [proof of proposition (3.5)]

We show that the two value functions can cross at most at two points (intersections with different slope). Hence, for a fixed q there are three possible cases: no crossing, one crossing, and two crossings. Then, we map the three possible cases with the level of q.

Prelimiaries ($P7 - P8$ are proved at the end)

$P7$: for $w \leq \widetilde{w}^c$ it exists a unique q_0^* invariant in w such that:

$$\begin{aligned} q &< q_0^* : V^e(w) < V^c(w) \\ q &> q_0^* : V^e(w) > V^c(w) \\ q &= q_0^* : V^e(w) = V^c(w) \end{aligned}$$

with $q_0^* = \frac{\left[\pi+(1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} - [\pi+(1-\pi)\gamma^{\sigma-1}]}{\left[\pi+(1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} - 1}$

$P8$ for $\widetilde{w}^c < w < \min(\widetilde{w}^e, \widehat{w}^c) : q^*(w)$ is stricly increasing. Let $\widetilde{q} = q(\min(\widetilde{w}^e, \widehat{w}^c))$

For the rest of the proof will will assume $q \neq q_0^*$ and describe at the end the special case $q = q_0^*$

By $P1$ and $P4 - P5$, the graphs of $V^e(w)$ and $V^c(w, q)$ can intersect in zero, one or two points

Let us first characterize the possible cases and show then how they apply to different values of q

case a: one intersection

By $P4 - P5$ at the unique intersection point w_h $\frac{\delta V^e(w,q)}{\delta w} < \frac{\delta V^c(w)}{\delta w}$

case b :two intersections

Let's call w_l and w_h, the two point of intersection where they intersect twice

By $P4 - P5$, at w_h, $\frac{\delta V^e(w,q)}{\delta w} < \frac{\delta V^c(w)}{\delta w}$. Then at w_l, $\frac{\delta V^e(w,q)}{\delta w} > \frac{\delta V^c(w)}{\delta w}$. which implies:

$$\begin{aligned} w &< w_l : V^e < V^c : \text{covered banking is optimal} \\ w_l &< w < w_h : V^e > V^c : \text{exposed banking is optimal} \\ w &> w_h : V^e < V^c : \text{covered banking is optimal} \end{aligned}$$

case c: no intersection

By $P4 - P5, V^c > V^e$ for all level of w

Consider how these cases apply for different values of q

By $P7$ and $P4 - P5$, when $q < q_0^*$, case a applies

By $P8$ and $P4 - P5$ when $q_0^* < q < \widetilde{q}$, case b applies

By Proposition (3.4), when $q > \pi$, case c applies

Now we use $P2$ to demonstrate by continuity which cases apply to the remaining range $q \in]\widetilde{q}, \pi]$.

Using the envelope thereom $\frac{dV^e(q,w)}{dq} = -(\pi u(c_E) + (1-\pi) u(c_L)) + u(c_R) < 0$

Thus, as q continuously decreases, $V^e(w,q)$ continuously increase while the $V^c(w)$, remains invariant.

By continuity $\exists!$ q_1 such that:

$$\begin{aligned} q_1 &< q < \pi : \textit{case } c \text{ applies} \\ \widetilde{q} &< q < q_1 : \textit{case } b \text{ applies} \\ q &= q_1 : V^b(w,q) \text{ and } V^c(w) \text{ are tangeant} \end{aligned}$$

By a similar reasoning, when there are two intersections points $w_l, w_h : \frac{\delta w_l}{\delta q} > 0; \frac{\delta w_h}{\delta q} < 0$.

By $P7 - P8$, $\min(\widetilde{w}^e, \widehat{w}^c) < w_l < w_h$

special case: $q = q_0^*$:

By $P7$ for $w < \widetilde{w}^c : V^e(w, q_0^*) = V^c(w)$

When $w \geq \widetilde{w}^e$ the analysis is as above and over $]\widetilde{w}^e, \infty)$ and by $P4 - P5$, case b applies on $]\widetilde{w}^e, \infty)$

Having shown the relative position of $V^c(w)$ and $V^e(w,q)$ for all values of q and all values of w, the proof is complete.

proofs of $P7 - P8$

Let : $\Delta(w,q) = V^e(w,q) - V^c(w)$

for $w \leq \underline{k}$: since $k^c = k^e = w$

$$\Delta(w,q) = V^e(w,q) - V^c(w) = [V^e(1,q) - V^c(1)]w^{\beta(1-\sigma)}$$

then:

$$\Delta(q^*, w) = 0 \Leftrightarrow [V^e(1,q) - V^c(1)] = 0$$

then $q^* = q_0^*$ is a constant, independent of w

for $\underline{k} < w \leq \widetilde{w}^c$

$V^c(w) = \pi u(c_E) + (1-\pi)u(c_L)$ and as $c_E = w - \underline{k} + \gamma h(\underline{k})$ and $c_L = c_E/\gamma$:

$$V^c(w) = u(w - \underline{k} + \gamma h(\underline{k}))[\pi + (1-\pi)\gamma^{\sigma-1}]$$

$V^e(w,q) = (1-q)(\pi u(c_E) + (1-\pi)u(c_L)) + qu(w - \underline{k} + \gamma h(\underline{k})$

and $c_L = c_E/\gamma^{1/\sigma}$

then:

$$V^e(w,q) = (1-q)u(c_E)[[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}] + qu\left(w - \underline{k} + \gamma h(\underline{k})\right)$$

but also:

$$\begin{aligned} c_{run} &= \pi c_E + (1-\pi)\gamma c_L \\ c_{run} &= c_E\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right] \\ c_E &= \left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{-1}(w - \underline{k} + \gamma h(\underline{k})) \end{aligned}$$

then:

$$\begin{aligned} V^e(w,q) &= \left((1-q)\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma-1} + q\right)u(w - \underline{k} + \gamma h(\underline{k})) \\ &= \left((1-q)\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} + q\right)u(w - \underline{k} + \gamma h(\underline{k})) \end{aligned}$$

And at $q = q^*$

$$V^e(w,q) = V^c(w)$$

then by substituting it is clear that q^* does not depend on w :

$$\left((1-q^*)\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} + q^*\right)u(w - \underline{k} + \gamma h(\underline{k})) = [\pi + (1-\pi)\gamma^{\sigma-1}]$$

then:

$$q_0^* = \frac{\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} - [\pi + (1-\pi)\gamma^{\sigma-1}]}{\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} - 1}$$

<u>for $\widetilde{w}^c < w \leq \min(\widetilde{w}, \widehat{w}^c)$</u>

$V^c(w) = u(w - k + \gamma h(k))[\pi + (1-\pi)\gamma^{\sigma-1}]$

$V^e(w,q) = \left((1-q)[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}](\pi + (1-\pi)\gamma^{1-1/\sigma})^{\sigma-1} + q\right)u(w - \underline{k} + \gamma h(\underline{k}))$

so q^* :

$$\begin{aligned} u(w - k + \gamma h(k))[\pi + (1-\pi)\gamma^{\sigma-1}] &= \left((1-q^*)\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} + q^*\right)u(w - \underline{k} + \gamma h(\underline{k})) \\ \frac{u(w - k + \gamma h(k))}{u(w - \underline{k} + \gamma h(\underline{k}))} &= \frac{\left(-q^*\left(\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} - 1\right) + \left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma}\right)}{[\pi + (1-\pi)\gamma^{\sigma-1}]} \end{aligned}$$

with $\left[\pi + (1-\pi)\gamma^{\frac{\sigma-1}{\sigma}}\right]^{\sigma} > 1$

As w increases, k increases in the covered banking solution but remains constant in the exposed solution, thus $u(w - k + \gamma h(k))$ increases by less than $u(w - \underline{k} + \gamma h(\underline{k}))$ because $(\gamma h'(k) - 1) < 0)$.

Then the LHS goes down and, to restore equality, the RHS will have to go down, which necessarily implies that q has to increase.

$$\frac{\delta q^*}{\delta w} > 0$$

C The dynamics of wealth of a banking economy [proof of proposition (4.1)]

We start by showing that the growth rate of the economy with the optimal banking system is strictly decreasing in two steps. First, we show that the growth rate is decreasing within covered and exposed banking. Second, we show that the growth rate decreases when there is a change in banking regime.

Proof. ■

Step A: We prove that the growth rates within a covered banking system and within an exposed banking system are strictly decreasing

Region A: $1+g^{c,e}(w) = \frac{F^{c,e}(w)}{w} = \frac{(1-\beta)}{\beta}\left(1-\lambda^c(1-\gamma)\right) f'(w) \Rightarrow g^{c,e}(w)$ is strictly decreasing;
$1+g^{run}(w) = \frac{F^{run}(w)}{w} = \frac{(1-\beta)}{\beta}\gamma f'(w) \Rightarrow g^{run}(w)$ is strictly decreasing

region B: substituting for the expression of λ

$1+g^e(w) = (1-\beta)\left(1-(\lambda^*(1-\gamma)+\lambda^*)+(1-\lambda^*)(1-\gamma)\frac{w}{\underline{k}}\right)\frac{f'(\underline{k})}{w} \Rightarrow g^e(w)$ is strictly decreasing

$1+g^c(w) = (1-\beta)\left(1-(\pi(1-\gamma)+\pi)+(1-\pi)(1-\gamma)\frac{w}{\underline{k}}\right)\frac{f'(\underline{k})}{w} \Rightarrow g^c(w)$ is strictly decreasing
$1+g^{run}(w) = (1-\beta)\gamma\frac{f'(\underline{k})}{w} \Rightarrow g^{run}(w)$ is strictly decreasing

Region C,D:

$1+g^{e,c}(w) = (1-\beta)\frac{f(k^{e,c}(w))}{w}$
$1+g^{run}(w) = (1-\beta)\gamma\frac{f(k^e(w))}{w}$

covered: using f.o.c, $g'(w) = \frac{k'(w)h''(k(w))(1-\beta)\left(\frac{1-\pi}{\beta\pi}\right)^2}{\left(\frac{1-\pi}{\pi}\frac{h'(k)}{\beta}+1\right)} < 0$

exposed: $g'(w) < 0 \Leftrightarrow \beta w k'(w) < k \Leftrightarrow \beta w\frac{k'(w)}{k} < 1$

We show that $\beta w\frac{k'(w)}{k} < 1$

using f.o.c, (i) $\beta w\frac{k'(w)}{k} = \frac{\beta\frac{1-\pi}{\pi}\frac{w}{h(k)}}{\frac{1-\pi}{\pi}\beta\left(\frac{1}{h'(k)}+\frac{w-k}{h(k)}\right)+kB}$ with $B = \frac{(1-\beta)h'(k)}{k}\left(\frac{(1-\gamma)h'(k)+1-u'(x)}{1-\gamma h'(k)}\right)\left(\frac{x(\pi x+(1-\pi)\gamma)}{u'(x)\gamma(1-\pi)+\pi x h'(k)}\right)$ and $x = \frac{C_E}{C_L}$

Observe that $\left(\frac{1}{h'(k)}+\frac{w-k}{h(k)}\right) = \left(\frac{w+k(\beta^{-1}-1)}{h(k)}\right) > \left(\frac{w}{h(k)}\right)$ and $B > 0$, therefore $(i) \Rightarrow \beta w\frac{k'(w)}{k} < 1 \Leftrightarrow g'(w) < 0$

run: $1+g^{run}(w) = \gamma(1+g^e(w)) \Rightarrow g^{run}(w)$ is strictly decreasing

Step B: we prove that when there is a change in banking regime at w_l and w_h : $g(w_l)^+ < g\left(w_l\right)^-$ and $g(w_h)^+ < g\left(w_h\right)^-$

Proof. at w_h there is a switch from an exposed system to an covered system then: ■

$g(w_h)^+ < g\left(w_h\right)^- \Leftrightarrow k^e(w_h) > k^c(w_h)$

$k^e(w_h) > k^c(w_h) \Leftrightarrow \frac{\delta V^c}{\delta k}\Big|_{k=k^e(w_h)} > 0$

After some algebra: $\frac{\delta V^c}{\delta k}\Big|_{k=k^e(w_h)} > 0 \Leftrightarrow q < \pi$ which is true as $q = q^*(w_h) < \pi$

$=> k^e(w_h) > k^c(w_h) \Leftrightarrow g(w_h)^+ < g\left(w_h\right)^-$

Proof. w_l, by a similar argument, $g(w_l)^+ < g\left(w_l\right)^-$ ■

Step C: We show that $F^c(w)$, $F^e(w)$, and $F^{^{run}}(w)$ have a unique fixed point

Using l'Hopital rule, $\lim\limits_{w_{t-1}\to 0} \frac{F^{^{c,e,run}}(w)}{w} = \lim\limits_{w_{t-1}\to 0} F^{b\prime}(w) = \infty$

The optimal choice of capital is bounded above by $\overline{k}$, then $\lim\limits_{w_{t-1}\to\infty} \frac{F^{c,e,run}(w)}{w} = 0$

Therefore since the growth rate of the economy with the optimal banking system is stricly decreasing, $F^c(w)$, $F^e(w)$, and $F^{^{run}}(w)$ have a unique fixed point $\bar{w}^c$, $\bar{w}^e$and $\underline{w}^e$ respectively. Hence:

- $\bar{w}^c$is the unique stable steady state for an economy with covered banking (deterministic growth),
- $(\bar{w}^e, \underline{w}^e)$ is an absorbing range for an exposed economy with exposed banking (stochastic growth)

D Parameters

The parameters used for simulations are:

Factor productivity	$A = 3$
Capital share	$\beta = .4$
Liquidity needs	$\pi = .4$
Liquidation value	$\gamma = .5$
Risk Aversion	$\sigma = 2$

Table 1 : Real Income Per Capita and Systemic Banking Crises[1]

Income Quartile	Number of systemic banking crises[2]	Partition of crises
Q1	6	18.8%
Q2	9	28.1%
Q3	11	34.4%
Q4	6	18.8%
Total	32	100%

1/ average 1975-1998 GDP per Capita, 1995 US dollars.

2/ source: Caprio and Klingebiel (2003)

Figure 1: Fiscal Cost of Banking Crises (%GDP)

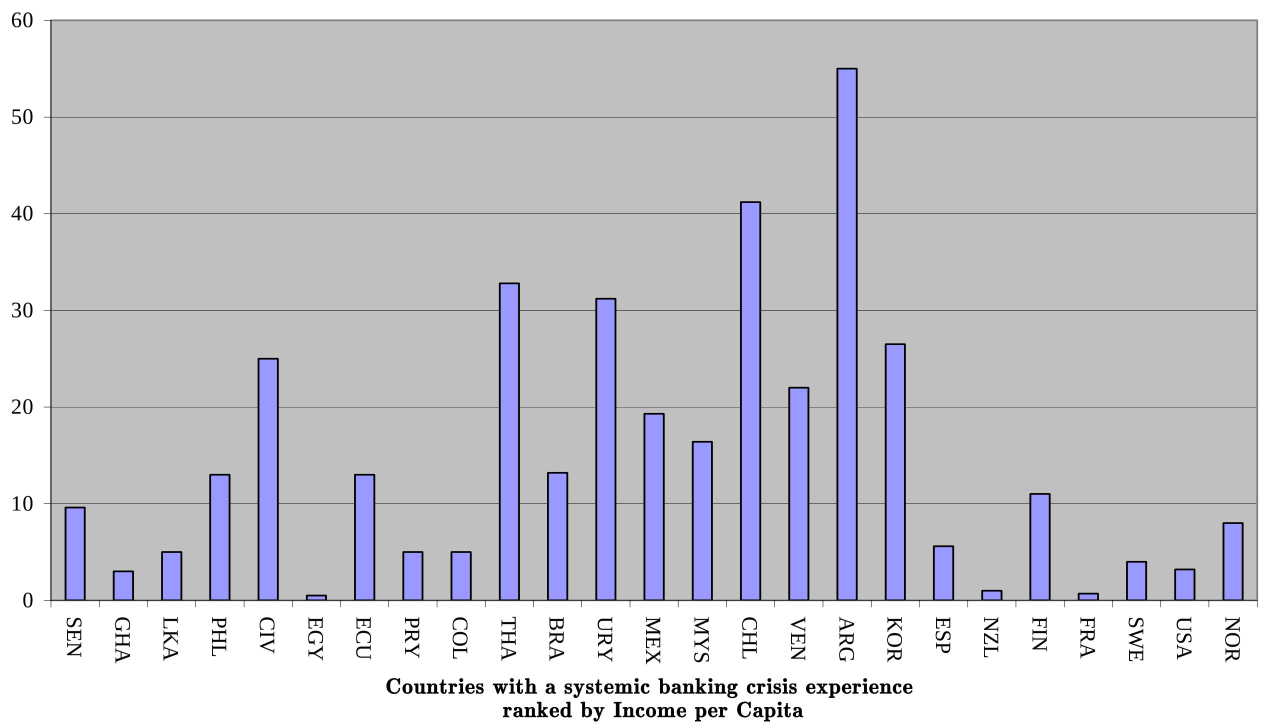

Figure 2: Timing

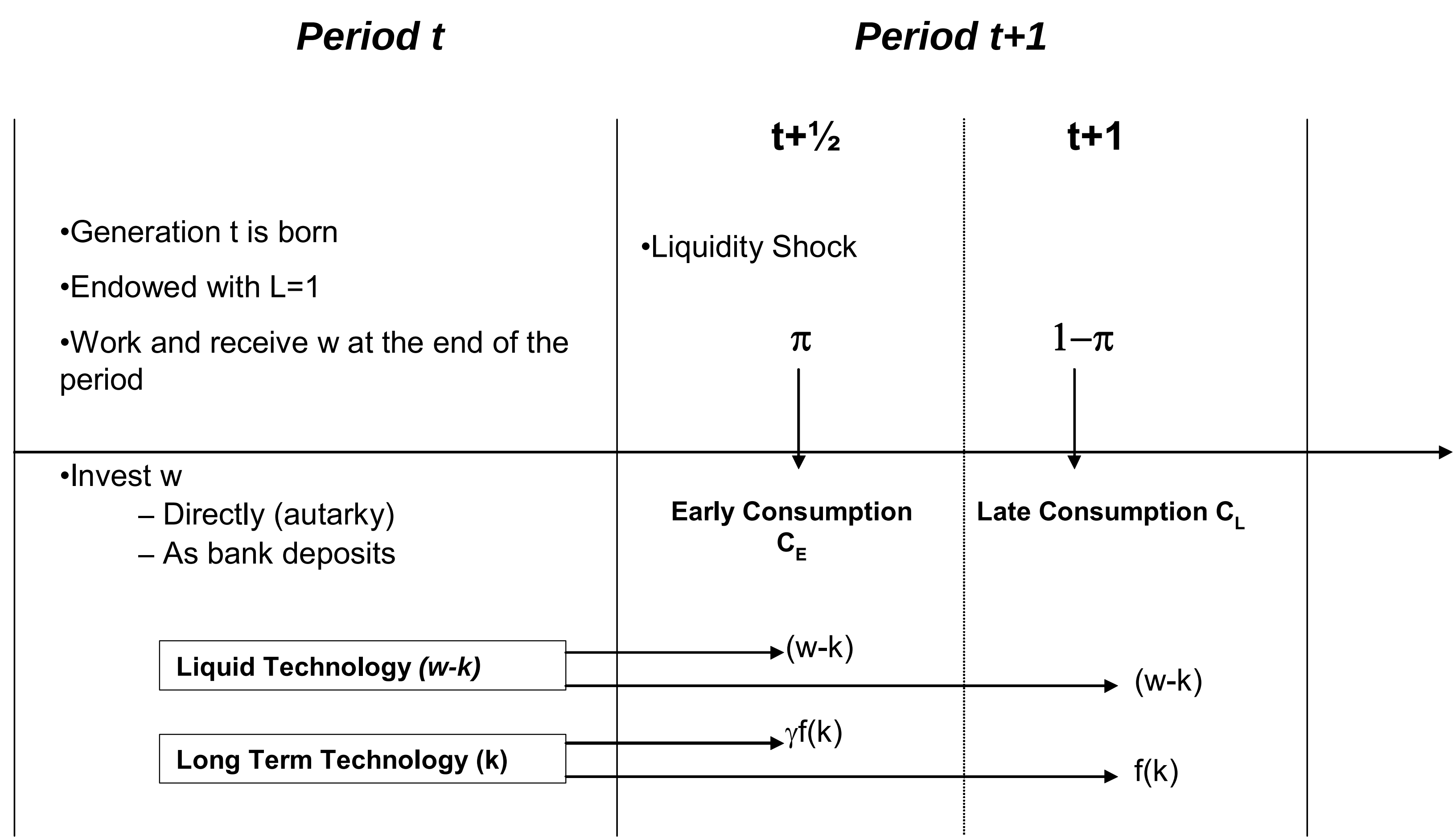

Figure 3: Marginal Returns

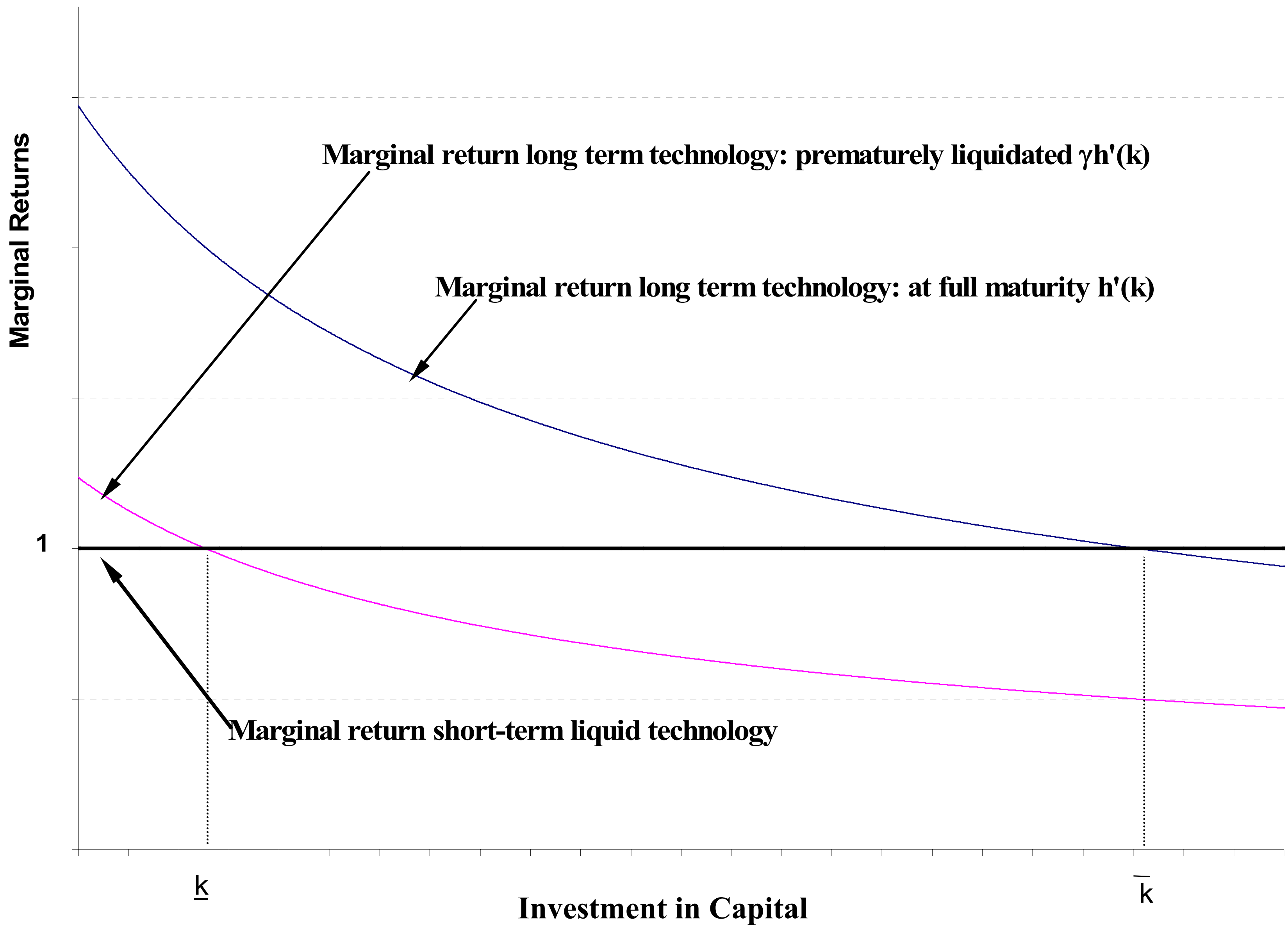

Figure 4a :THE BEST COVERED BANKING SYSTEM

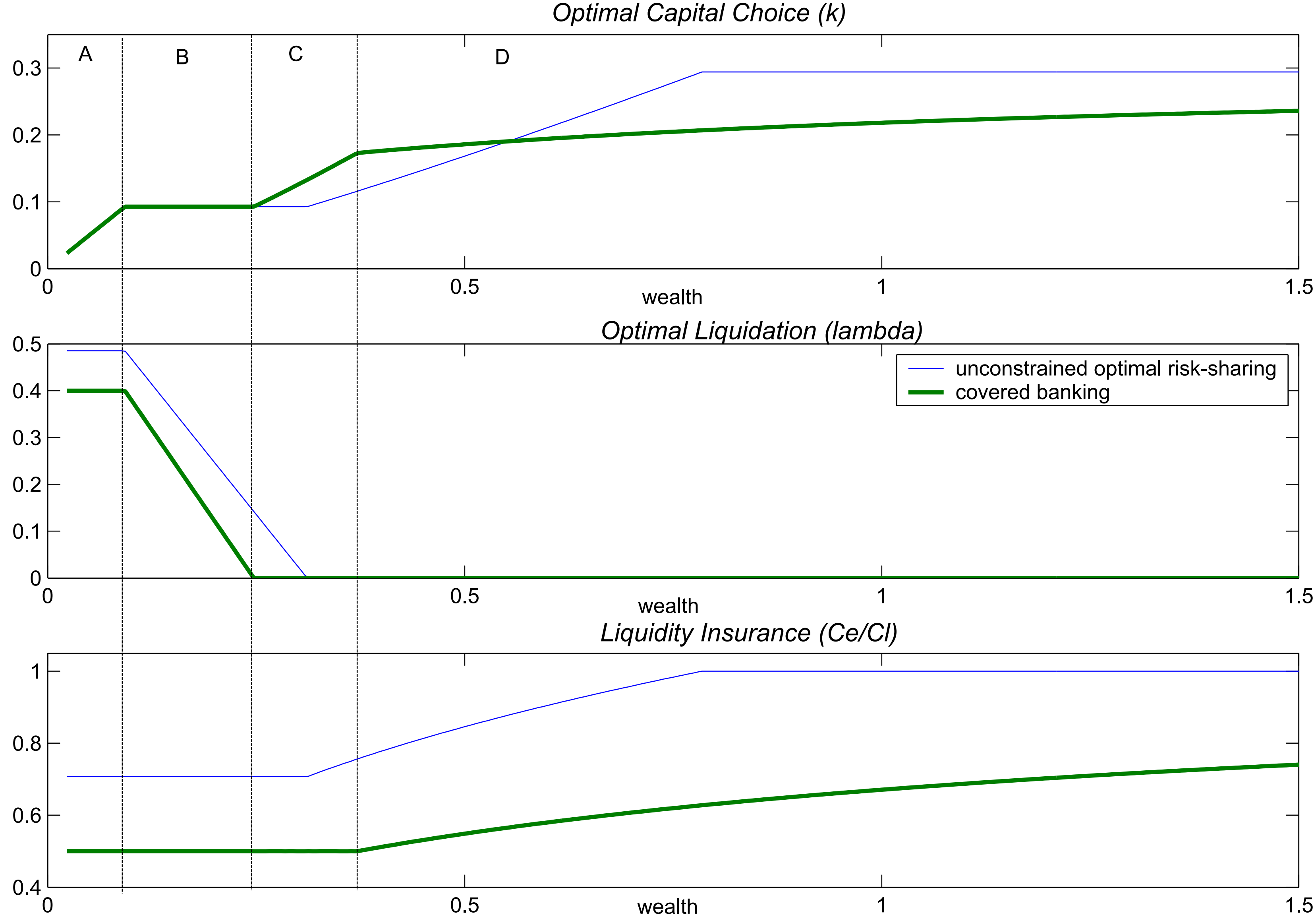

Figure 4b :THE BEST EXPOSED BANKING SYSTEM

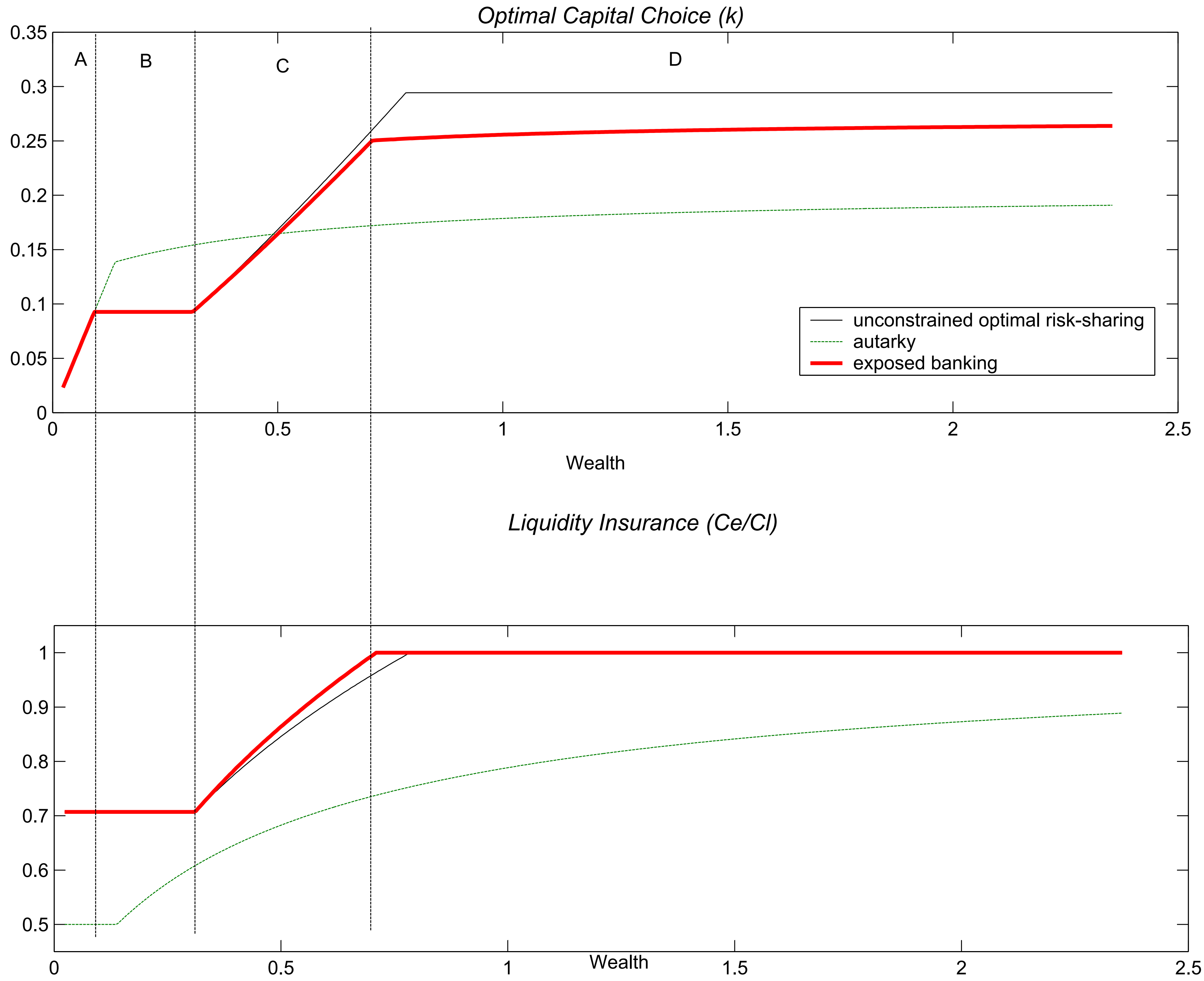

Figure 5: The Optimal Banking System

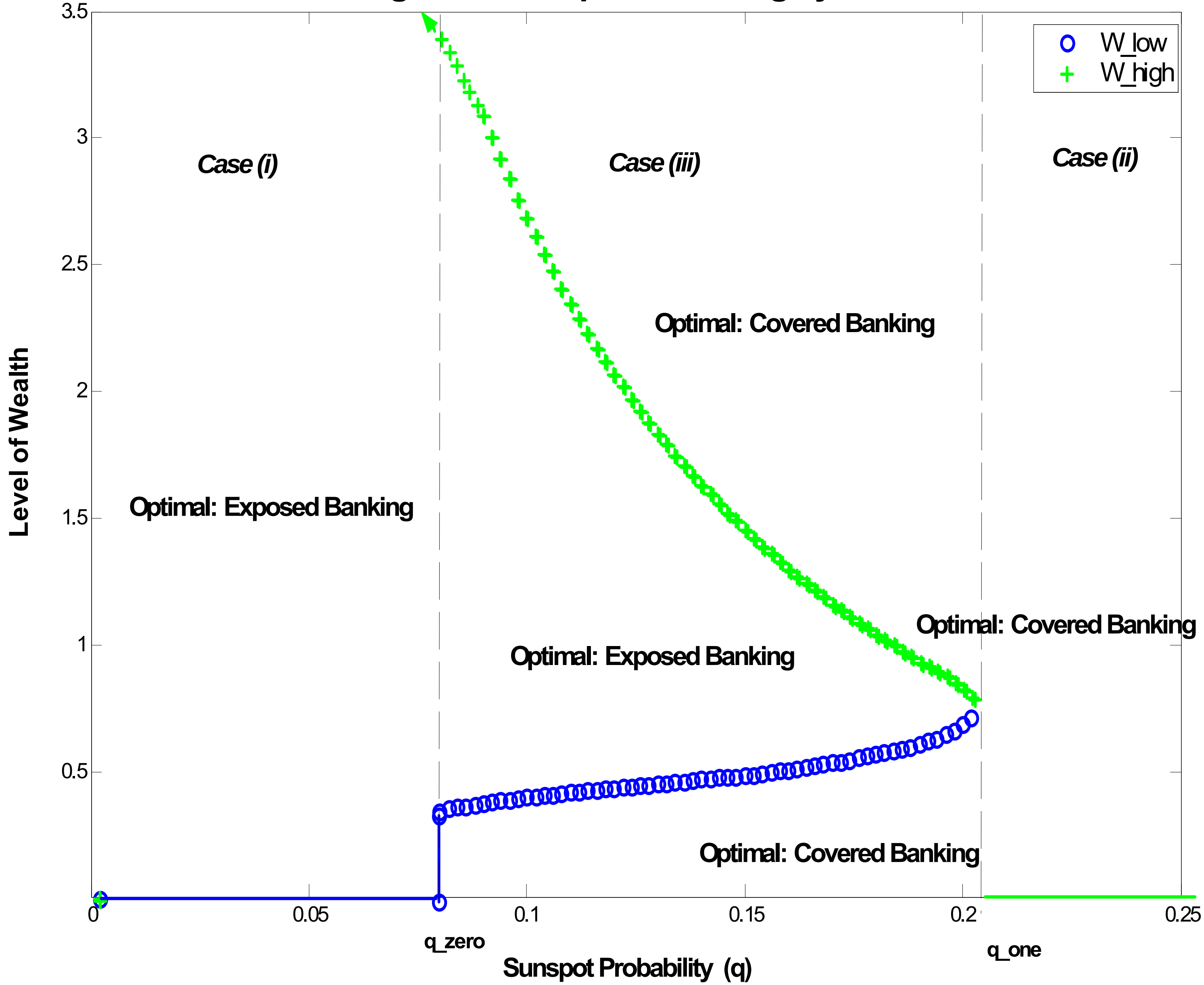

Figure 6: Optimal Banking System and the Growth Dynamics

(probability of run : 0.07)

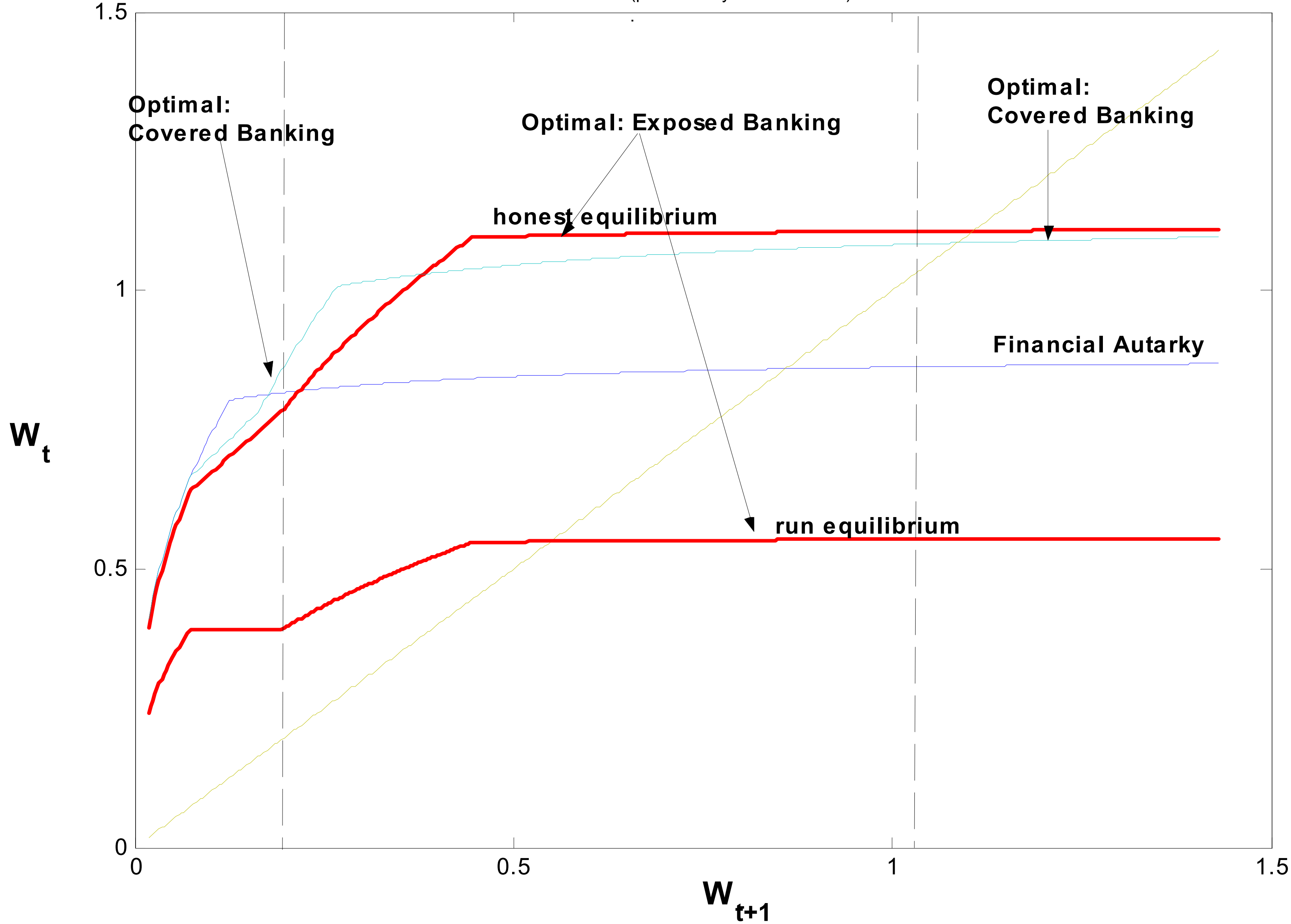

Figure 7: Relative Output Loss and the Level of Wealth